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Prediction of Single Sphere Motion in Newtonian and Non-Newtonian State Fluids

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Abstract: The motion of particles such as spheres immersed in fluids represents an idealization of various industrial processes. Such as, sedimentation and fluidized suspensions, lubricated transport, or general unit operation processes. The motion of spheres falling through static fluids was studied. The objective was to study the different flow patterns and to predict the terminal falling velocity and drag coefficient of particles settling in Newtonian and non-Newtonian (shear-thinning) fluids. The velocity of a falling sphere was measured as a function of time and sphere density. Particles of different sizes and densities, beside different types of fluids such as engine oil, glycerine, and kerosene representing non-Newtonian (shear-thinning)] were used. Derived experimental data for solid spheres falling through Newtonian and non-Newtonian fluids were reported, using fluid properties and hydrostatics bench apparatus. Empirical equations were formulated and developed for predicting drag coefficient and terminal falling velocity of solid spheres falling through stagnant fluids.

Keywords: *Hydrodynamics; Drags; Particle motion; Solid sphere; Terminal velocity*.

1. INTRODUCTION

The hydrodynamic properties of particles motions are very important in numerous industrial applications. These properties (viscosity, size, shape, density, etc) affect particle motions on fluids.[1] The fluid dynamic drag on a sphere and the terminal settling velocity of a single spherical particle in static fluid are of interest in numerous fields such as unit operation processes. The practical application without complicating features; may be suitable for study of particles motions. The free terminal velocity is a fundamental engineering parameter which must be determined in order to predict the hydrodynamic behaviour of particles within a flow. The flow past a sphere in a confined region is encountered in various applications such as falling ball viscometer, hydrodynamic chromatography, membrane transport, and hydraulic transport of coarse solids in pipes.[2] Furthermore, numerous fluids of industrial importance display shear-thinning characteristics which are conveniently approximated by the simple power law models, some of these non-Newtonian fluids are; polymer melts, polymer solutions, food emulsions, suspensions and biological fluids [3].

The objective of this work is to collect experimental data of falling particles of different sizes and densities. Different types of Newtonian and Non-Newtonian fluids were used as media for free particle falling. It is intended also, to improve the well known published correlations for drag and terminal settling velocity that have been in use, based on the collected data and other data obtained from many previous investigations. The behaviour of a particle undergoing acceleration or retardation has been the subject of a very large number of investigators. The results obtained by different workers were not consistent. It is although shown that the drag factor is often related, not only to the Reynolds number, but also to the particle diameters, the distance travelled by the particle since the initiation of the motion, the hydrodynamic equations which control the motion of particle in fluid [4].

2. MATERIALS AND METHODS

Data has been collected for the falling velocities of single spheres in Newtonian and non-Newtonian fluids using glass, steel, plastic and rubber balls of different diameters. The properties of the spheres are shown in Table 1. Also different types of fluids were used; distilled water, (the oil type) oil, glycerine and kerosene. Physical properties of these fluids are shown in Table 2.

Table 1. Properties of solid spheres used in the tests

Sphere	Weight(g)	Diameter (m)×10^{-2}	Density (kg/m^3)
Glass(1)	9.10	1.89	2574.00
Glass(2)	4.90	1.55	2513.00
Plastic	5.40	1.64	2338.00
Rubber	8.03	2.19	1460.00
Steel(1)	1.00	0.65	6954.00
Steel(2)	0.50	0.49	8116.00
Steel(3)	0.42	0.45	7800.00

Table 2. Properties of fluids tested

Fluids	Density (kg/m^3)	Temperature ($^\circ$C)
Glycerine (100%)	1255.00	31.0
Distilled water	0995.34	31.0
Lub Oil (grade (50))	0880.00	31.0
Kerosene	0817.00	31.0

2.1 Experimental Procedure

Single particle was placed on the fluid surface at the centre of the tube and left for free falling. This usual technique was to attain the terminal falling velocity in a short period. A digital stop watch of precision (0.01 s) was used to measure the time required by the particle to cover the distance (100 cm) marked on the tube. The recorded times were only considered when the difference between two successive reading did not exceed 1%. This was to ensure that the particle has attained its terminal falling velocity during the free falling. Each run was repeated at least three times for each particle and an average was taken.

2.2 Measurements of Solid Velocities Falling in Newtonian and Non-Newtonian Liquid

Data collected for the falling velocities of single spheres in Newtonian and non-Newtonian fluids were calculated. From the measurements an average velocity for each combination of sphere – fluids was computed. Using the average velocities determined the solid – fluids properties were estimated. The parameters of interest (i.e. V_t, C_D, Re) were then computed and recorded. Also apparent viscosity (dynamic viscosity) of fluids was calculated; by method of falling sphere [5]. Then estimated shear rates and shear stresses of fluids were calculated using the power law fluids (parameter n and k). The most common approach taken by previous investigators for predicting the terminal velocity was through the use of standard Newtonian relationships (C_D–Re) using modified (non-Newtonian or generalized) Reynolds number. For power law fluids, the generalized Reynolds number was defined by, [6-7].

$$\mathrm{Re}_{gn} = \frac{\rho V^{2-n} d^n}{k} \tag{1}$$

Where, k and n are two empirical curve-fitting parameters, are known as the fluid consistency coefficient and the flow behaviour index; respectively. For a shear-thinning fluid, the index may have any value between 0 and 1. The smaller the value of n, the greater is the degree of shear-thinning. Power-law fluid behaviour and data correction for the shear rate ($\frac{V_t}{d_p}$) with which the shear rate reflects the definition of particle Reynolds number (Re_p) shown in equation (2).

$$\mathrm{Re}_p = \frac{d V_t \rho_f}{\mu_a} \tag{2}$$

2.3 Dimensionless Parameters

Several dimensionless parameters were used to describe the hydrodynamic equations. They include:

$$C_D = \frac{4\Delta\rho g d}{3\rho V_t^2} \tag{3}$$

$$d^* = \left[\frac{3C_D \mathrm{Re}_P^2}{4}\right]^{1/3} = d\left[\frac{\rho_f(\rho_s-\rho_f)g}{\mu^2}\right]^{1/3} \tag{4}$$

$$V_t^* = \frac{\mathrm{Re}_P}{d^*} = \left[\frac{4\,\mathrm{Re}_P}{3C_D}\right]^{1/3} = V_t\left[\frac{\rho_f^2}{\mu(\rho_s-\rho_f)g}\right]^{1/3} \tag{5}$$

When used non-Newtonian fluids the dimensionless include apparent viscosity μ_a and the power law parameter n and k. [7].

$$d^* = (Ar)^{\frac{2-n}{2+n}} = \left[\frac{3}{4}C_D \mathrm{Re}^{\frac{2}{2-n}}\right]^{\frac{2-n}{2+n}} = d\left(\frac{g\Delta\rho}{\rho}\right)^{\frac{2-n}{2+n}}\left(\frac{\rho}{k}\right)^{\frac{2}{2+n}} \tag{6}$$

$$V^* = \left[\frac{\mathrm{Re}}{\left(\frac{3}{4}C_D\right)^n}\right]^{\frac{1}{2+n}} = V\left[\frac{\rho^{n+1}}{g^n k \Delta\rho^n}\right]^{\frac{1}{2+n}} \tag{7}$$

where,

Ar: Archimedes number.

$$Ar = C_D \mathrm{Re}^{\frac{2}{2-n}} = \frac{4gd^{\frac{2+n}{2-n}}}{3k^{\frac{2}{2-n}}\rho^{\frac{2}{n-2}}}\left(\frac{\Delta\rho}{\rho}\right)$$

$\mathrm{Re} > 1$ (8)

Two sources were used for the behaviour of fluids [8]. Table 3 shows the Range of Re and power law parameter.

Table 3. Comparison of Re and Power Law Parameter range with literature

Source	Re	K ($kg.s^n/m^2$)	n
Non-Newtonian			
Case Study	12 - 370	0.028 – 0.788	0.587 – 0.971
Kelessidis [7]	1 – 64	0.020 – 0. 270	0.750 – 0.920
Miura [7]	4 - 770	0.170 – 0.590	0.560 – 0.780
Newtonian			
Case Study	1 - 325	0.035 – 0.126	1.0
Kelessidis [7]	16 - 1270	0.010 – 0.060	1.0

3. RESULTS AND DISCUSSION

Depending on the experimental results of the k and n values all the values of Reynolds number of the particles (Re_p) were converted. The Archimedes' number was plotted versus the particles Reynolds number for non-Newtonian fluids. The regression analysis of these plots showed that the values of n and k are 0.779 and 0.403 respectively.

For all the spheres used in the experimental work and power-law liquid combination, the Archimedes number (Ar) can be evaluated. Then the particle Reynolds number can then be expressed in terms of Ar and n [9] as follows:

$$\mathrm{Re} = aAr^b \qquad (9)$$

where,

$$a = 0.1\exp\left(\frac{0.51}{n} - 0.73n\right) \text{ and } b = \frac{0.954}{n} - 0.160$$

The expression for (**a**) is erroneously printed without the multiplication factor of 0.1, and using the equation with the wrong coefficients gives always a considerably underestimated velocity [9]. So by the same approach propose similar expressed to evaluate experimental data. Suggested simple empirical equation describes the relation between Ar and Re for non-Newtonian fluids, Fig. 1 as follows:

$$\mathrm{Re} = aAr^b \qquad (10)$$

where,

$$a = 0.01\exp\left(\frac{1.42}{n}\right) \text{ and } b = \frac{0.881}{n} - 0.160$$

Covered the range $27 \le Ar \le 11775$ and $1 \le Re \le 481$, with R^2 = 0.9915. Experimental data describe the parameters of interested, Ar, Re_p, Re and C_D predicted by equation (10), When used the value of n = 0.779 and substituted in equation (9) and (10) for the two constants a, b. it was Found that a = 0.108 and b = 1.065. Compared with predicted a = 0.062 and b = 0.971. the error 8% for b and 42% for a, always the value of a corrected for given density of fluid (ρ),density difference (Δρ), sphere diameter (d), consistency index (k) and the value of power law behaviour (n). Fig. 2 shows experimental data of drag coefficient related to the Reynolds number for non-Newtonian fluids. Fig. 3 shows predicted Re related to C_D measured.

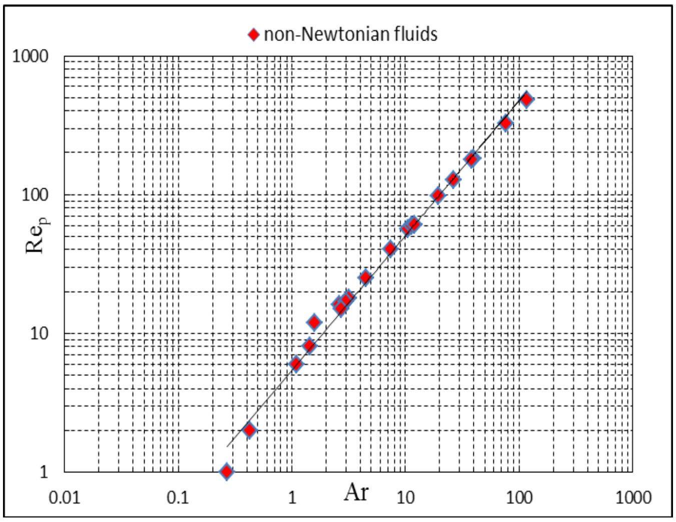

Fig .1. Experimental data, (Archimedes' number verse particles Reynolds number).

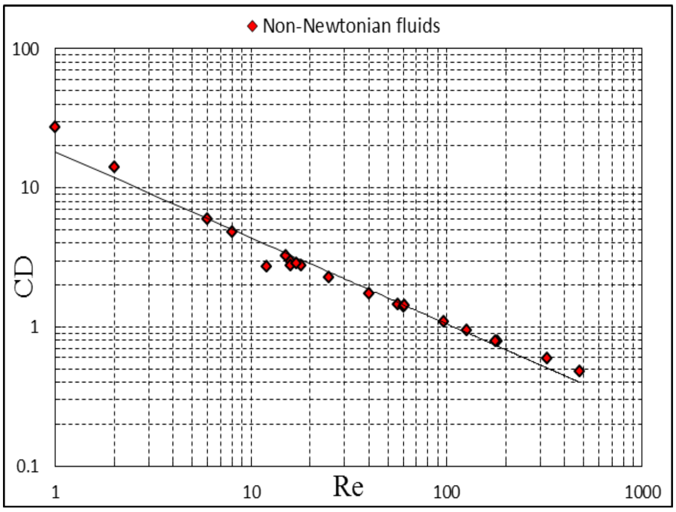

Fig. 2. Behaviour of non-Newtonian fluids

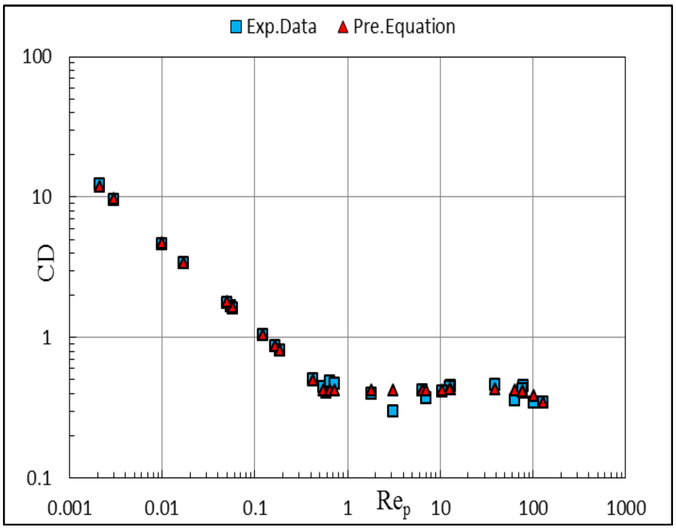

Fig. 3. Experimental drag compared with prediction for Newtonian and non- Newtonian fluids

3.1 Drag Coefficient Formula Proposed

The approach suggest plotting the drag coefficient versus particles Reynolds number on a logarithmic paper .then compare them with prediction by other investigators. Through the region flow of Newton's law the parameters of interest V_t, C_D, Re_p, are estimated. By used nonlinear regression of 28 data points, selected form 49 point. The best fitted supporting by trials and error methods [10-11]. Then we obtained simple an empirical equation to predict drag coefficient. The equation covered range of Reynolds number $2 \leq Re_p \leq 129635$ for Newtonian and non-Newtonian fluids. C_D is predicted with two terms, the first term can be considered as an extended Stokes' law applicable approximately for $Re_p < 424$ as,

$$C_D = \frac{18.62}{Re_P^{0.6}} \quad \text{For} \quad 2 \leq Re_p \leq 424 \tag{11}$$

The second term for slight deviations from the Newton's law for high particles Reynolds number,

$$C_D = 0.42 + \frac{6\,Re_P}{E^7} - \frac{9\,Re_P^2}{E^{12}} \quad \text{For } 550 \leq Re_p \leq 129635 \tag{12}$$

Figures 2 and 3 show the experimental data fitted of non-Newtonian behaviour and the prediction of drag coefficient compare with experimental. Figures 4-6 show also comparison of drag measured and predicted with (SDC) and other investigators [9].

3.2 Root Mean Square

Equation of the general form given in the four most recent correlations were sought, with the parameters determined by a local minimization of the sum of the squared errors, Q, defined as [12].

$$RMS = \sqrt{\frac{Q}{N}} \tag{13}$$

where $$Q = \left(\sum \log C_{D\exp} - \log C_{D_{Pre}}\right)^2$$

So for correlation the predicted equations (11) and (12) to estimate drag coefficient find that, the RMS_C_D value, computed and shown in Table (4).

3.3 Data and Prediction Comparison

Fig (5) shows the plotting of the non-Newtonian data for drag coefficient C_D versus the Reynolds number Re. The Newtonian as well as the non-Newtonian data is plotted in Fig (6). Data from other investigators are also plotted, both non-Newtonian data and Newtonian data [9].

Table (4) RMS_C_D values.

	Newtonian and non-Newtonian	Non-Newtonian only
RMS_C_D	0.041	0.062

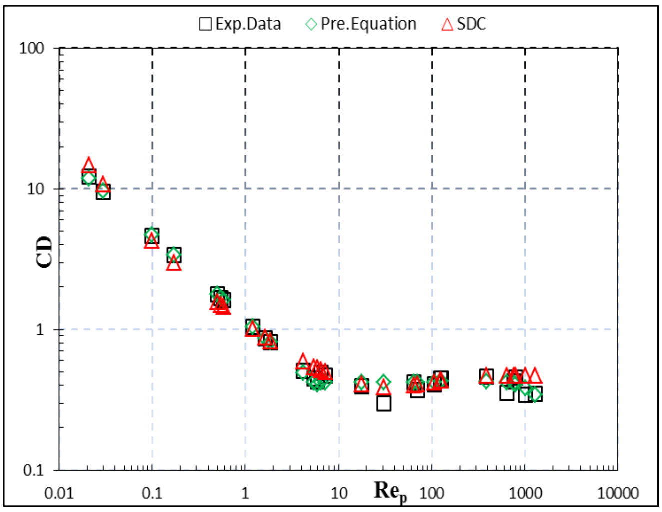

Fig.4. Comparison of experimental data of drag related to Reynolds No

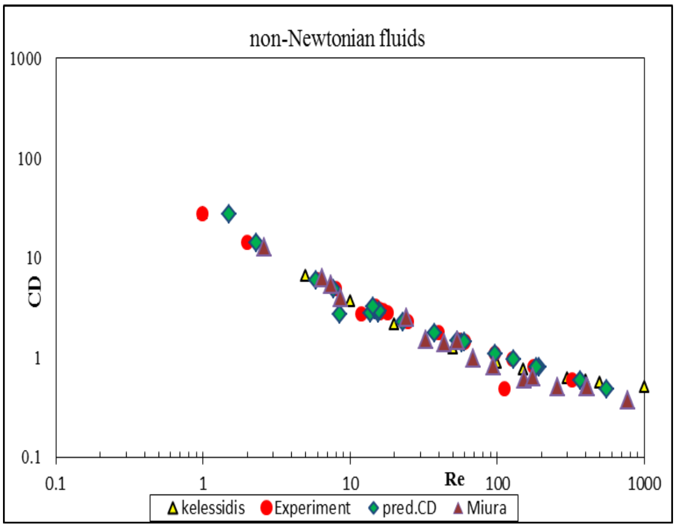

Fig. 5. Comparison of experimental measurements with regression equations results, non- Newtonian data.

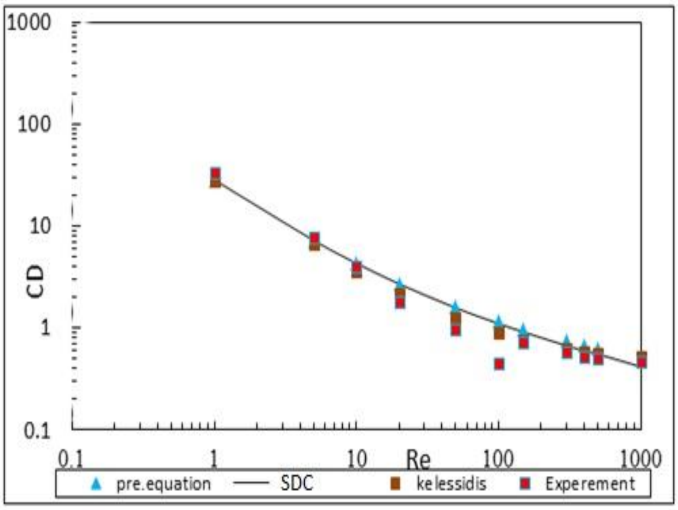

Fig. 6. Comparison of experimental measurements with regression equations results, non-Newtonian and Newtonian data.

3.4 Correlation Terminal Setting Velocity

On the other hand from experimental data analysis obtained through different particles diameter used and the Grace Method for the solution of the settling-velocity equation [12-13]. For terminal falling velocity as function of diameter, using two dimensionless parameters, (V_t^* and d^*) shown in Figures (7) and (8). The proposed formulas for Newtonian and non-Newtonian fluids are as follows:

$$\log(V_t^*) = 3E^{-7}\log(d^{*3}) - 5E^{-4}\log(d^{*2}) + 0.35\log(d^*) - 2.89 \tag{14}$$

$$0.74 \le V_t^* \le 142 \quad \text{and } 4 \le d^* \le 935$$

For non-Newtonian fluids,

$$\log(V_t^*) = 1.30 - 0.35\log(d^*) + 0.07\log(d^{*2}) - 0.0031\log(d^{*3}) \tag{15}$$

$$0.77 \le V_t^* \le 14 \text{ and } 4 \le d^* \le 65$$

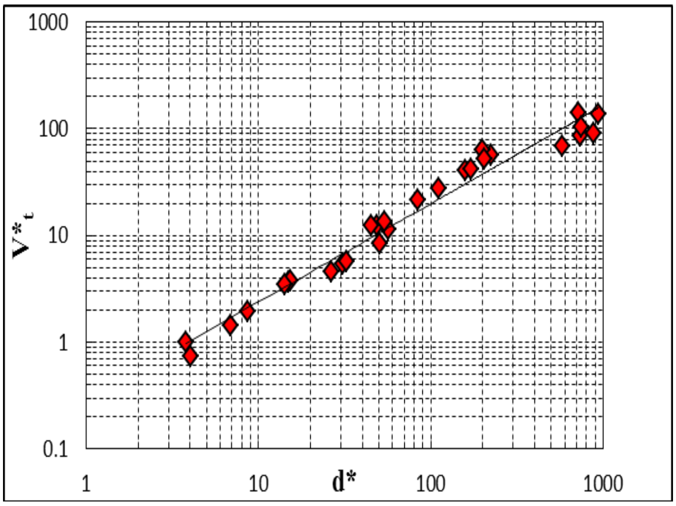

Fig . 7. Dimensionless terminal velocity, V^*_t, as a function of dimensionless particle diameter, d^*, for spheres falling in Newtonian fluids

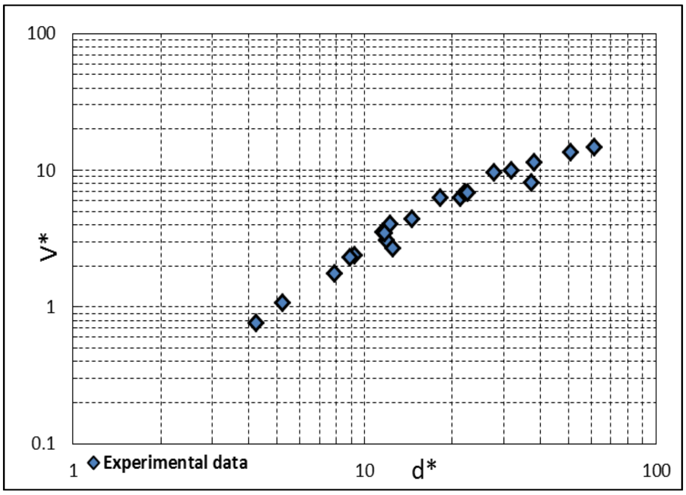

Fig .8. Dimensionless terminal velocity, V^*_t, as a function of dimensionless Particle diameter, d*, for spheres falling in non- Newtonian fluids.

The purpose of most of the cited sphere drag experiments was to develop correlations describing either the drag coefficient or terminal velocity of a sphere. The results presented in the previous section are of great importance of drag coefficient prediction. Fig. 6 shows C_D measured and predicted compared to SDC, very small variation for all Re_p values less than 10^4, that means the empirical equation can be extended to predict C_D for Newtonian and shear-thinning fluids.

For all experimental data, indicates that the RMS_C_D errors are around 0.06, shown in table (4). The deviation in experimental data of non- Newtonian compared with Kelessidis indicated that, are smaller in the range of low Re_p (stokes region) and variation in intermediate Re_p (creeping flow), are shown in Figs 5 and 6. Foregoing analysis has indicated that the terminal falling velocity of solid spheres through stagnant non-Newtonian shear thinning fluids can be predicted with engineering accuracy from various proposed correlations for Newtonian fluids. This can be achieved provided the apparent viscosity of the fluid is used, evaluated at a shear rate given as the ratio of the falling velocity to the sphere diameter.

4. CONCLUSIONS

Hydrodynamic equations of particles motion were investigated on this study. The behaviour of drag and terminal velocity on spherical particles flow in Newtonian and non-Newtonian fluids were explored. This provided information such as the drag coefficient for particles of various sizes and density; which are much required parameters in CFD programs for the prediction and modelling of particle flow. It is recommended that the temperature effect must be considered, for fluids properties, because it has great effective on the dynamic viscosity which extended to shear rate and behaviour of particles motion.

The further work can be used for other types of fluids, such as visco-elastic and gaseous fluids.

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Nomenclature

Ar	Archimedes' number	dimensionless
a	Parameter assigned for equation (9)	dimensionless
b	Parameter assigned for equation (10)	dimensionless
C_D	Drag coefficient	dimensionless
d	Particle diameter	cm
d*	Dimensionless particles diameter	dimensionless
K	Consistency index	dimensionless
n	Fluid flow behaviour index	dimensionless
R_e	Reynolds number	dimensionless
R_{egn}	Generalized Reynolds number	dimensionless
R_{ep}	Particle Reynolds number	dimensionless
V	Solid terminal velocity	m/s
V_t	Particle terminal velocity	m/s
V_t*	Dimensionless terminal velocity	dimensionless

Greek letters

$\Delta\rho$	Density difference, $(\rho s-\rho)/\rho$	g/cm^3
ρ_s	Solid density	g/cm^3
ρ_f	Fluid density	g/cm^3
μ	Fluid viscosity	g/cm.s
μ_a	Apparent viscosity	g/cm.s

Subscripts

a	Apparent
exp	Experimental
gn	Generalized
t	Terminal
p	Particle
pre	Predicted
s	Solid
*	Dimensionless

Abbreviations

RMS	Root Mean Square
SDC	Standard Drag Curve
CFD	Computational Fluids Dynamics