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High Voltage Stress and Equipotential Mapping Using FEM Method

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Abstract: The aim of this paper is to study the voltage stress and the equipotential mapping for a sample of 33kV porcelain insulator, which is used widely in the Sudanese national grid. The available modeling and analysis package COMSOL multi-physics was used, and a limited number of experiments using the high voltage laboratory facility at University of Khartoum was attempted to compare the simulation and experimental results. The simulation predicts that 30 kV/cm ionization (for air at standard conditions) occurs at about 105 kV, while experimentally ionization started as a visible corona discharge at about 101 kV peak .

Keywords: *Comosol/M; Dirichlet boundary condition; Neumann boundary condition; Finite element method; Equipotential Mapping*

1. INTRODUCTION

The voltages stresses on power system arise from various over voltages, either external voltages which are associated with lighting discharges and are not dependent on the voltage of the system, or internal over voltages which are generated by changes in the operating conditions of the system such as switching operations, a fault on the system or fluctuation in the load or generation. In designing the insulation of the system, the two areas of specific importance are, determination of the stress which the insulation must withstand and, determination of the response of the insulation when subjected to those voltage stress [1]. Before any engineering analysis may be under taken for a power system, it is necessary to abstract from the physical object numerical models. Numerical models are descriptions in the form of algebraic equations or more usefully, computer programs. The first step for developing numerical models is to formulate differential equations that describe all interactions in the system; these equations are solved in a specific region with a set of boundary conditions for a desired variable. In recent years several numerical methods (FDM, FEM, BEM and CSM) for solving the general two- or three-dimensional field (Laplace`s and Poisson`s equations) have been available and used whenever the analytical methods are difficult or impossible to solve.

Finite Element methods (FEM) are the most efficient and popular computational methods for solving partial differential equations (PDES), and this is due to the fact that FEM have many advantages over many other numerical methods; Finite element methods can work on complex irregular geometries, they use non-uniform meshes to reflect solution gradation and they can construct high-ordered approximations. A finite element method utilizes a variational problem that involves an integral of the differential equations over the problem domain; the domain of the problem is divided into sub-domains called finite elements and the solution of the problems is approximated by simple polynomials functions on each element, these polynomials have to be pieced together so that the resulting approximate solution would have an appropriate degree of smoothness over the entire domain, then the variational integral is evaluated as the sum of the contributions from each finite element in an approximate solution given by piece-wise polynomials defined throughout the domain. Certain steps in formulating a finite element analysis of a physical problem are common to all such analyses, whether structural, heat transfer, fluid flow, electrostatic field or some other problem. These steps are embodied in commercial finite element software packages (some are mentioned in the software paragraph). These steps are described as follows [2]:

Pre-processing*:* The pre-processing step is, quite generally, described as defining the model and includes:

- Define the geometric domain of the problem.
- Define the element type(s) to be used
- Define the material properties of the elements.
- Define the geometric properties of the elements (length, area, and the like).

- Define the element connectivities (mesh the model)
- Define the physical constraints (boundary condition
- Conditions).
- Define the loadings.

Solution: During the solution phase, finite element software assembles the governing algebraic equations in matrix form and computes the unknown values of the primary field variable(s). The computed values are then used by back substitution to compute additional, derived variables, such as relative permittivity, reaction forces, and heat flow.

Post-processing: Analysis and evaluation of the solution is referred to as post-processing. Post-processing software contains sophisticated routines used for sorting, printing and plotting selected results from a finite element solution.

2. FIELD EQUATIONS

A. Electrical Field and Potential Distributions Calculation

For applications of ac having extra low frequency as 50/60 or dc voltages, problems may be considered as an electrostatic field problem and therefore the electric and magnetic field components may be considered independently of each other, and calculations made on the basis of static field concepts. In the case of electrostatic and (also quasi-static) fields only Maxwell electric field equations (flux divergence) are considered. One simple way for electric field calculation is to calculate electric potential distribution. Then the electric field distribution is directly obtained by the negative gradient of electric potential distribution [3]. In an electrostatic field problem, electric field distribution can be written in Cartesian system of coordinates as follows [4]:

$$E = -\nabla V \quad (1)$$

From Maxwell's equation

$$\nabla E = \frac{\rho}{\varepsilon} \quad (2)$$

where ρ is resistivity in $\Omega/m.$, ε is material dielectric constant. $\varepsilon = \varepsilon_0 \varepsilon_r$, ε_0 is free space dielectric constant (8.854 $\times 10^{-12}$ F/m) , ε_r is relative dielectric constant of dielectric material. Placing equation (1) into equation (2) Poisson's equation is obtained.

$$\varepsilon \nabla.(\nabla V) = -\rho \quad (3)$$

without space charge $\rho = 0$, Poisson's equation becomes Laplace's equation

$$\varepsilon \nabla.(\nabla V) = 0 \quad (4)$$

B. Fem Analysis of The Electric Field Distribution

The two- dimensional problem for which Laplace's or Poisson's equations applies is:

$$\nabla \phi^2 = \frac{\partial \phi^2}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \begin{cases} 0 \text{ ....(Laplacian region)} \\ -F(x,y)\text{.......(Poissonian region)} \end{cases} \quad (5)$$

where ϕ is electric potential and F(x,y) = ρ / ε for electrostatic fields within a medium of permittivity ε and containing distributed charge of density $\rho(x, y)$. The field problem is given within an x-y plane, the area of which has to be limited by given boundary conditions, by contours on which some field quantities are known.

Poisson's equation is more general than Laplace's equation, and Poisson's equation in terms of the Cartesian coordinate system is defined as :

$$\nabla u^2 = \frac{\partial u^2}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = g(x, y) \quad (6)$$

-

For a two–dimensional problem the boundary conditions are

$$u = \bar{u} \text{ and } \frac{\partial u}{\partial n} = \bar{q} \quad (7)$$

where u is the electrostatic potential in the x-y plane and $\bar{u}$ and $\bar{q}$ are known voltage and flux variables defining boundary conditions, and *n* is outward normal unit vector at the boundary [5]. Supposing that the domain under consideration does not contain any space and surface charges, a two-dimensional functional F(u) in the Cartesian system of coordinates can be formed as follows [6] :

$$F(u) = \frac{1}{2} \int_D \left[\varepsilon_x \left(\frac{du}{dx} \right)^2 + \varepsilon_y \left(\frac{du}{dy} \right)^2 \right] dxdy \quad (8)$$

where ε_x and ε_y are the x and y components of dielectric constant in the Cartesian system of coordinates and u is the electric potential. In case of isotropic permittivity distribution ($\varepsilon = \varepsilon_x = \varepsilon_y$) equation (8) can be reformed as**:**

$$F(u) = \frac{1}{2} \int_D \varepsilon \left[\left(\frac{du}{dx} \right)^2 + \left(\frac{du}{dy} \right)^2 \right] dxdy \quad (9)$$

Inside each sub-domain D_e a linear variation of the electric potential is assumed as described in (7)

$$u_e(x, y) = \alpha_{e1} + \alpha_{e2} + \alpha_{e3} \quad (e = 1,2,3,...n) \quad (10)$$

where $u_e(x,y)$ is the electric potential of any arbitrary point inside each sub-domain D_e. α_{e1}, α_{e2}, α_{e3} represent the computational coefficients for a triangle element e, n is total number of triangle elements.

The calculation of the electric potential at every node in the total network composed of many triangle elements was carried out by minimizing the functional F(u) , that is :

$$\frac{\partial F(u)_i}{\partial u_i} = 0 \quad ; i = 1,2, np \qquad (11)$$

where, np stands for the total number of nodes in the network. Then a compact matrix expression

$$\lfloor S_{ji} \rfloor \lfloor u_i \rfloor = \lfloor T_j \rfloor, i, j = 1, 2, ...np \qquad (12)$$

where $\lfloor S_{ji} \rfloor$ the matrix of coefficients is, $\lfloor u_i \rfloor$ is the vector of unknown potentials at the nodes and $\lfloor T_j \rfloor$ is the vector of free terms. By solving equation (12) the unknown potential can be found.

C. Boundary Value Problem (BVPs)

Boundary value problem is a system of ordinary differential or partial differential equations with solution and derivative values specified at more than one point. Most commonly, the solution and derivatives are specified at just two points (the boundaries) defining a two-point boundary value problem [6]. There are two type of boundary that are considered in the electrical field evaluation [7]:

1. Boundary between conducting and materials.
2. Boundary between different dielectric materials.

For electrical point view, one of the following conditions may be satisfied on the boundary of the first type [8]:

Dirichlet boundary condition: is a type of boundary condition, named after Johann Peter Gustav Dirichlet, which when imposed on an ordinary or a partial differential equation, specifies the values a solution needs to take on the boundary of the domain. If the electric potential of all points are known, and this is possible when the available conductor is connected to a fixed potential source, the boundary condition is a Dirichlet one.

Neumann boundary condition: is a type of boundary condition, named after Carl Neumann, which when imposed on an ordinary or a partial differential equation, specifies the value that the derivative of the solution is to take on the boundary of the domain, for example if the whole charge on the boundary surface is known, while the electric potential over different points is unknown. This is a case when the available conductor is not connected to a fixed potential (floating). Normally in such a case the whole charge is equal to zero, the boundary is Neumann one.

3. SOFTWARE

The finite element method is computationally intensive owing to the required operations on very large matrices. In the early years, applications were performed using mainframe computers, which, at the time, were considered to be very powerful, high-speed tools for use in engineering analysis. During the 1960s, the finite element software code NASTRAN was developed in conjunction with the space exploration program of the United States. NASTRAN was the first major finite element software code. It was, and still is, capable of hundreds of thousands of degrees of freedom (nodal field variable computations). In the years since the development of NASTRAN, many commercial software packages have been introduced for finite element analysis. Among these are ANSYS, ALGOR, and COSMOS/M. In today's computational environment, most of these packages can be used on desktop computers and engineering workstations to obtain solutions to large problems in static and dynamic structural analysis, heat transfer, fluid flow, electromagnetic, and seismic response [2]. In this paper, we used COMSOL Multiphysics 3.4 for analysis the electric field and potential distributions for a porcelain insulator.

COMSOL Multiphysics® [9] is an interactive commercial software package for modeling and simulating scientific and engineering problems based on partial differential equations. The modeling features of COMSOL Multiphysics allow you to simultaneously model any combination of phenomena in 2D and 3D, for all fields of engineering and sciences. The COMSOL graphical user interface includes functions for CAD modeling, physics or equation definitions, automatic mesh generation, equation solving, visualization, and postprocessing. Key features include: state-of-the-art solvers for simultaneous solving of an arbitrary number of coupled linear, nonlinear, and time-dependent PDEs. COMSOL Multiphysics allows for multiphysics modeling including electrostatic, thermal, structural, fluid, electromagnetic, chemical reactions, fuel cells and other components in a single, fully coupled model.

4. INSULATION MATERIAL

Electrical insulation is a medium or a material which when placed between conductors at different potentials permits only a small or negligible current in phase with the applied voltage to flow through. The term dielectric is almost synonymous with electrical insulation, which can be considered the applied dielectric. A perfect dielectric passes no conduction current but only capacitive charging current between conductors. Only a vacuum at low stresses between uncontaminated metal surfaces satisfies this condition.

The rang of resistivities of substances which can be considered insulators is from greater than 1020 Ω.cm downward to the vicinity of 106 Ω.cm, depending on the application and voltage stress.

The value of the relative dielectric constant (k) ranges from unity for vacuum to slightly greater than unity for gases at atmospheric pressure , 2- 8 for common insulating solids and liquids , 35 for ethyl alcohol and 91 for pure water and 1000 to 10000 for titanate ceramics [10] .

Ceramic insulators are made of ceramic materials which include porcelain and glass; their initial use precedes the construction of power systems. They were first introduced as components in telegraph networks in the late 1880s. There are a number of basic designs for ceramic insulators. Porcelain is used for the production of cap and pin suspension units, solid and hollow core posts, pin type, multi-cone and long rod insulators, and bushing housings. Glass, on the other hand, is used only for cap and pin suspension and multi-cone posts.

Table (1) Dielectric Permittivity (Relative Dielectric constant)

Material	K		K
Inorganic crystalline		**Classes**	
NaCl, dry crystal	5.5	Fused silica	3.8
$CaCO_2$ (av)	9.15	Coring 7740 (commom Lab. Pyrex)	5.1
Al_2O_2	10.0	**Polymer resins**	
MgO	8.2	**Nonpolar resins**	
BN	4.15	polyethylene	2.3
TiO_2	100	polystyrene	2.5-2.6
$BaTiO_2$crystal	4100	polypropylene	3.2
Muscovite mica	7-7.3	polytetelrafluoroethylene	2
Fluorophlogopite (synthetic mica)	6.3	**Polar resine**	
Ceramics		Polyvinyl chloride (rigid)	3.2-3.6
Alumina	8.1-9.5	Polyvinyl acetate	3.2
Steatite	5.5-7.0	Polyvinyl fluoride	8.5
Forsterite	6.2-6.3	Nylon	4-4.6
Aluminum silicate	4.8	Polyethylene terephthalate	3.25
Typical high-tension porcelain	6.0-8.0	Cellulose cotton fiber (dry)	5.4
Titanates	50-10000	Cellulose kraft fiber (dry)	4.9
Beryl	4.5	Cellulose cellophane (dry)	6.6
Zirconia	8-10.5	Tricyanoethyl triacetate	15.2
Magnesia	8.2	Epoxy resins unfilled	3-4.5
Class-bonded mica	6.4-9.2	Methylmethacrylate	3.6

Standard Handbook for Electrical Engineers

Porcelain and glass insulators are well established, as might be expected based on their long history of use [1].

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5. PHYSICAL PROBLEM

A. Insulator Geometry

A 33kV porcelain insulator was selected to simulate electric field and potential distributions; the basic design of the insulator is as follows: porcelain shed with different diameters having a relative dielectric constant of 6., cement having relative dielectric constant of 2.33. , and surrounding of the insulator is air having a relative dielectric constant of 1. A conductor is directly connected to the voltage source and run through the top groove; the porcelain is grounded by a metal rod screwed into the lower metallic thread. The total creepage distance is 685 mm. **Fig (1)** show the actual shape, **Fig (2)** show the cross section area of the insulator.

Fig. 1. Insulator

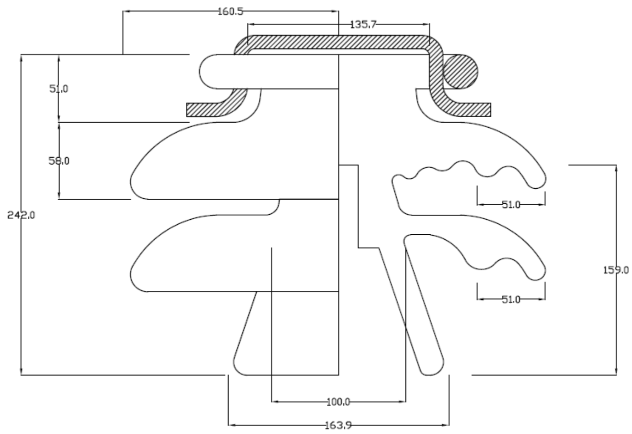

Fig. 2. Cross section

B. Simulation Using COMSOL Multiphysics

The physical problem which was studied and simulated is : 33 kV porcelain insulator the potential, electrical field and equipotential surfaces for the two dimensional field is to be determined for normal voltage and flash over voltage. Flash over is defined as a disruptive discharge through air or over the surface of solid insulation, between parts of different pollution or polarity by the application of voltage wherein breakdown path become sufficiently ionized to maintain an electric arc [11]. Discharge activity on the surface of a high voltage insulator is caused by the local electric field having a value higher than the ionization level of the ambient. This high electric field is the result of the applied voltage and the environmental conditions such as rain, pollution etc [12].

The solution of equation (5) depends on boundary conditions of the problem. When the problem is solved by FEM, all boundary conditions of the problem must be known, since the porcelain is unbounded, an artificial boundary is assumed as the boundary at infinity as shown in figure (3) according to this , boundary conditions are taken as :

V= V_0 volt on the conductor at the upper part of the porcelain and this is a Dirichlet boundary condition.
V=0 (ground) volt, on the lower part of the porcelain, and also a Dirichlet boundary condition.

$\frac{\partial V}{\partial n} = 0$, on all the other outer boundary, and this is a **Neumann boundary condition**.

n.(D_1-D_2) = 0 on the surfaces of the dielectric of the insulator as continuity . The normal vector **n** , points from medium (2) to medium (1).

6. AIR FLASHOVER OF A CLEAN DRY INSULATOR

If the electric stress in air at atmospheric pressure exceeds 2.6 kV/mm (measured value), ionization can occur depending on the gap configuration, flashover may follow. The power flashover voltage of a clean dry single Cap and Pin insulator with a 280mm creepage distance is 72 KV. In practical insulation systems, the solid material is stress in conjunction with one or more other material. If one of the material is, gas or air, the measured breakdown voltage will be influenced more by the weak medium than by the solid . On applying the voltage V between two electrodes with a slab between them, with homogeneous field a fraction of voltage appears across the ambient given by :

$$V_1 = \frac{Vd_1}{d_1 + d_2(\frac{\varepsilon_1}{\varepsilon_2})} \qquad (13)$$

A simple case when a gaseous dielectric is in series with a solid stress between two parallel plate electrodes. In equation (13) d_1 and d_2 represent the thickness of the media 1 (air) and media 2 (solid) , and ε_1 and ε_2 are their respective permittivities . The stress in the gaseous part will exceed that

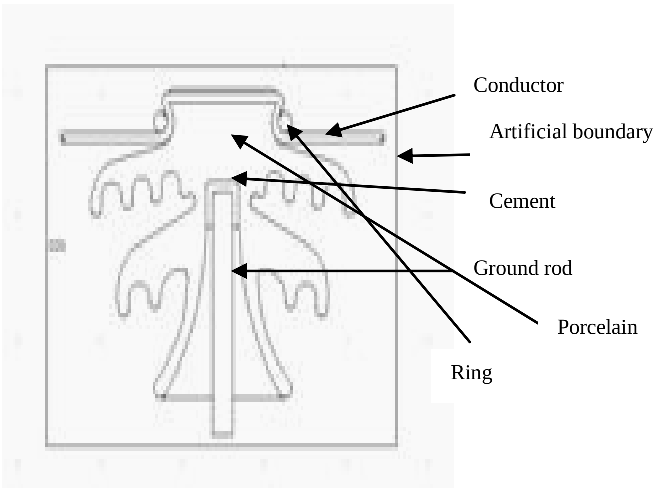

Fig. 3. Geometry of the sample

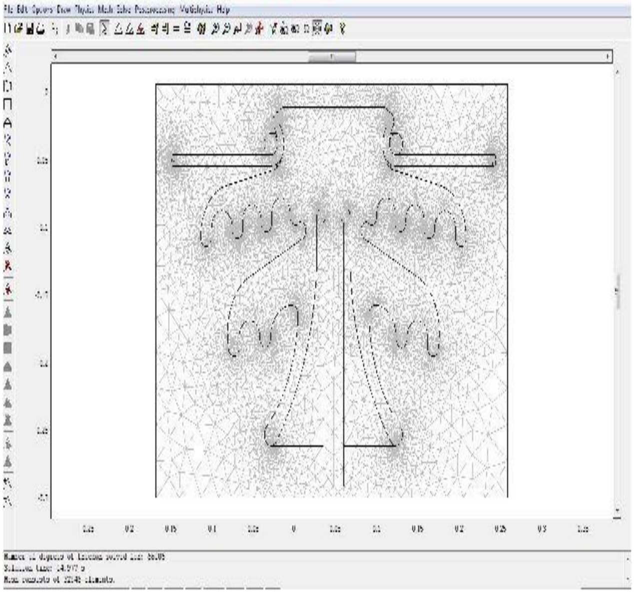

Fig. 4. Mesh (27kV peak)

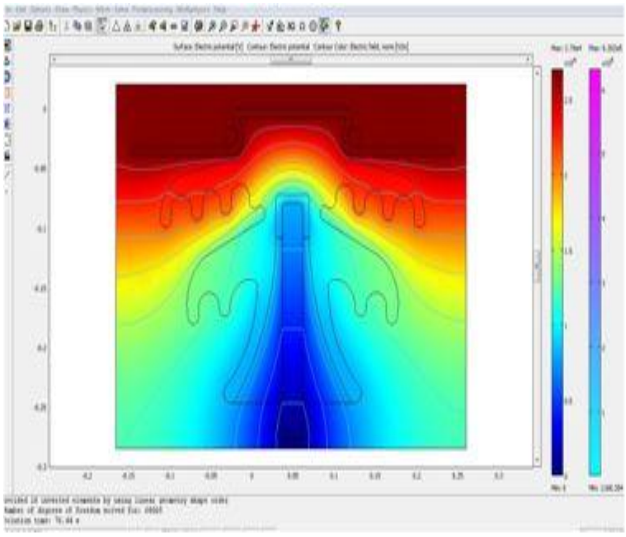

Fig. 5. Equipotential mapping (27kV peak)

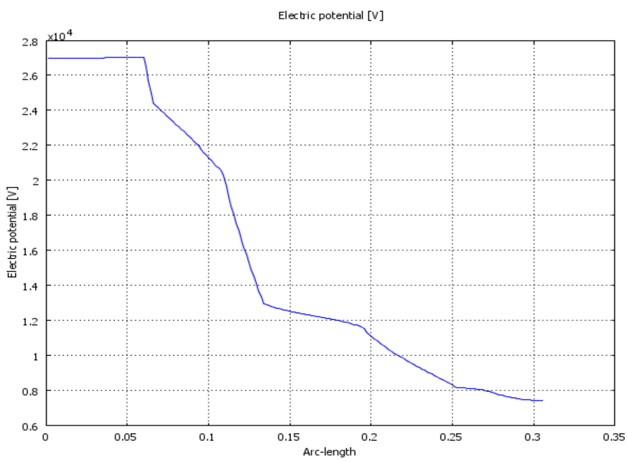

Fig. 6. Arc- length of potential (27kV peak)

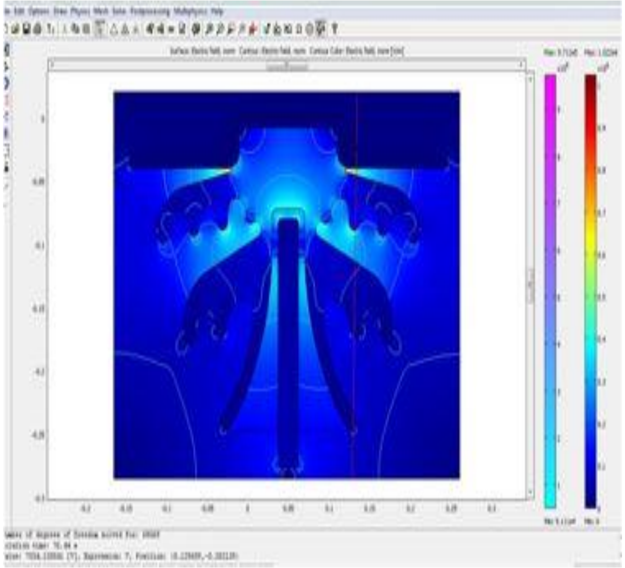

Fig. 7. Electric field

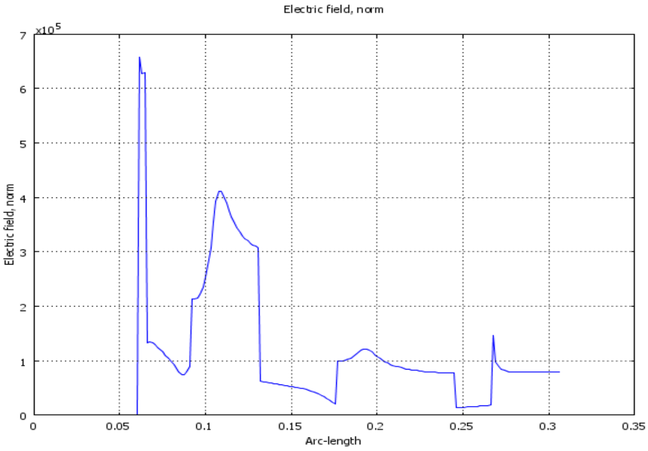

Fig. 8. Arc- length diagram for electric field (27kV peak)

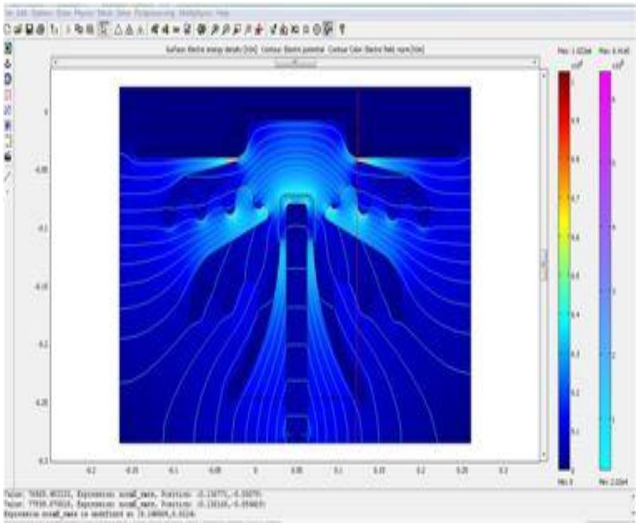

Fig. 9. Equipotential mapping and Electric field (27 KV)

of the solid by the ratio of permittivities. Consequently the ambient breaks down at a relatively low applied voltage.
The results obtained from FEM using COMOSL Multiphysics, is laid out in figures (3-14) for two cases:

First case:

A 27 kV peak voltage is applied first; to specify the normal voltage. Figures (3 to 9) show the mesh -which consists of 23946 node- , potential mapping, electric field and line diagram along the porcelain for the first case.

Second case :

A 50-120 kV is applied second, to specify the ionization voltage, resulting in this ionization voltage (max electric field = 30 kV/cm) occurring at around 105 kV figures (10-14) show mesh, potential mapping, electric field and line diagram when the voltage reaches 105 kV .
Figures (7,12) show clearly that the electric field (which is the gradient of the voltage) is smaller when the voltage lines are more spread out; in the conductor there is negligible voltage drop across its length, therefore there no electric field within it.

d . Experimental Work

Experimental field analysis techniques are methods in which the real system is tested using high voltage equipment, where the breakdown and flashover quantities are much easier to measure. For the sample insulator considered here laboratory tests are carried out in ordered to validate the results obtained from computer simulations.

The equipment of the insulator test consists of a single phase high voltage transformer (500/300000V) in the University of Khartoum, connected to the test piece through a system of conductors and glass insulators. The control of the system is through a console which allows powering the equipment and incorporates the necessary safety and monitoring equipment. A current-and voltage measuring devices is included (**Figs. 15 -18).**

The secondary voltage from the transformer (load voltage) and the leakage current was reported in **Table 2** by changing the voltage source by hand until the complete flash over voltage occurs and **Fig. 19** shows the relation between leakage current and load voltage. The Figure shows the insulator tendency of the leakage current to increase proportionally with the voltage across the insulator. **Fig. 20** shows the flash over on the surface and around the insulator caused by higher voltage (112.2 kV). It was noted, however, that a partial discharge on the insulator surface started to occur in the vicinity of 72 kV rms (corresponding to 102 kV peak). At the same time a disproportionate increase in leakage current was observed. The experimental results are in reasonable agreement with the simulation since ionization was predicted to start at the 100 kV region.

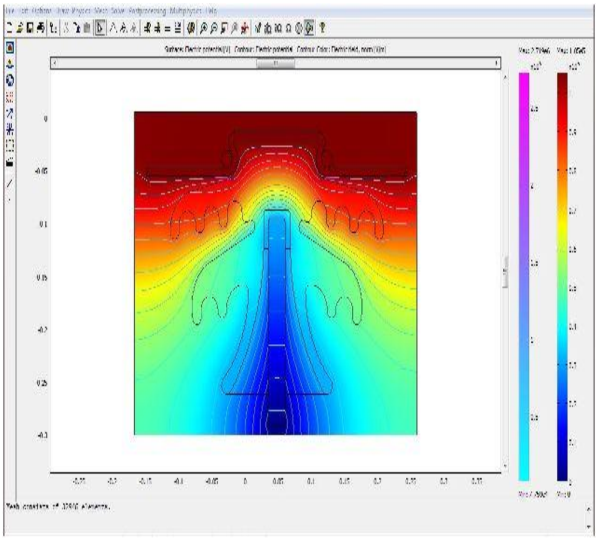

Fig. 10. Equipotential mapping (105 KV)

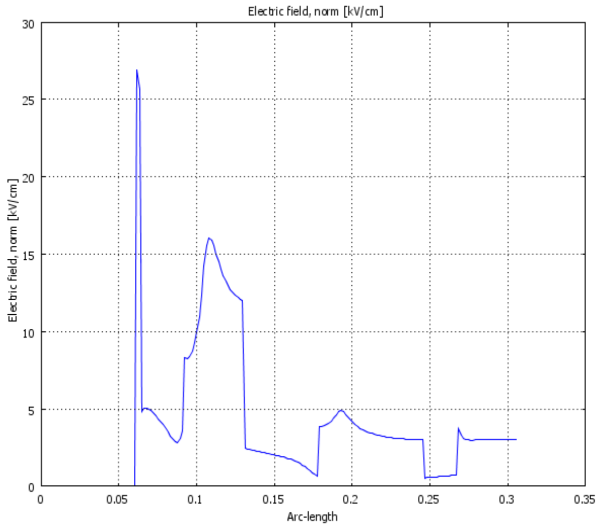

Fig. 13. Arc- length diagram for electric field (105 kV peak)

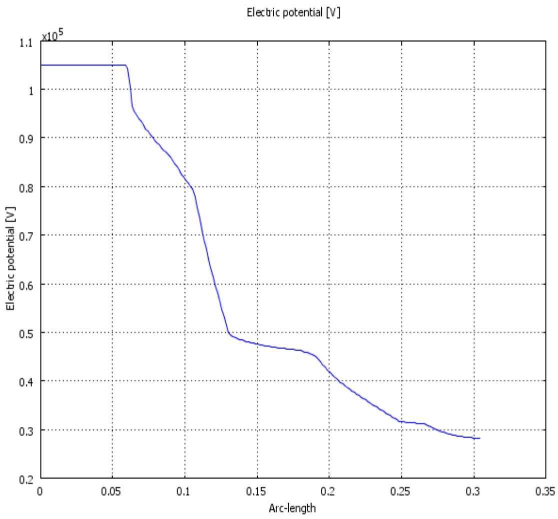

Fig. 11. Arc- length of potential (105kV peak)

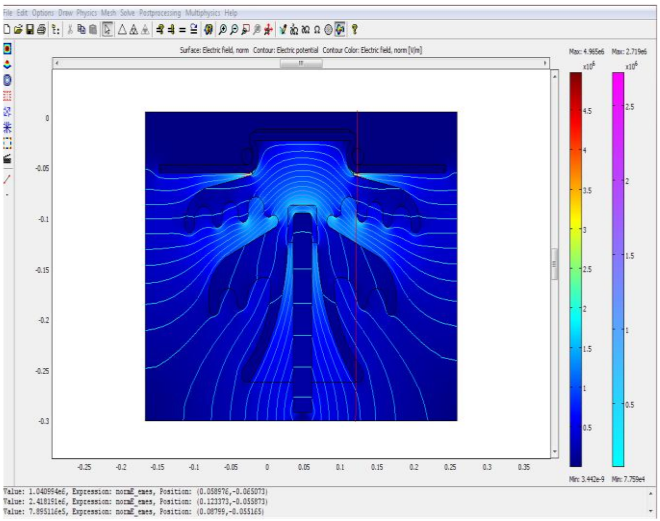

Fig. 14. Equipotential mapping and Electric field (105 KV)

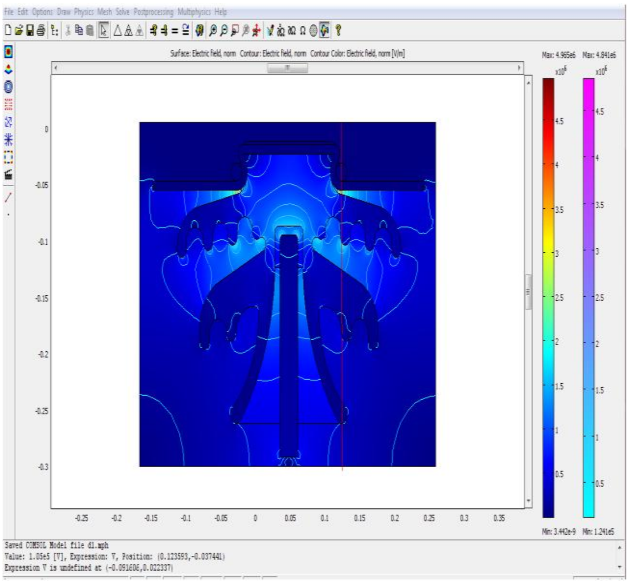

Fig. 12. Electric field (105 kV)

Table 2. Experiment results

Voltage (kV)	Leakage Current (A)	Comment
0	0	
60	0.0041875	
72	0.00804	Partial discharge begins
78	0.008375	
84	0.0085425	
90	0.00871	
112.2	0.011725	Complete Flash over

Fig. 15. Transformer

Fig. 16. Insulator

Fig. 17. Control

Fig. 18. Switches

Various components of high voltage test

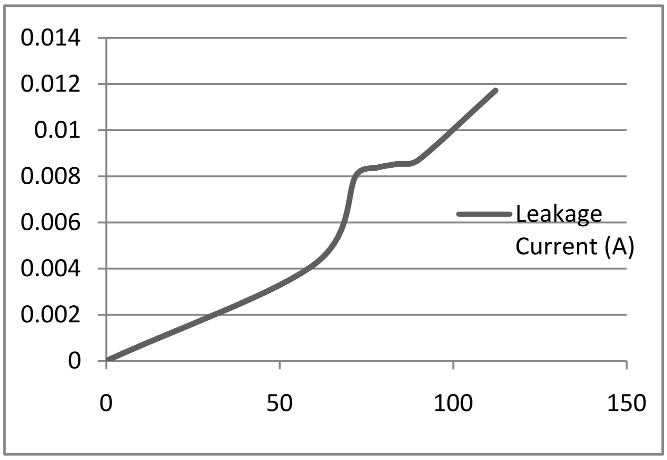

Fig. 19. Voltage Vs Leakage current

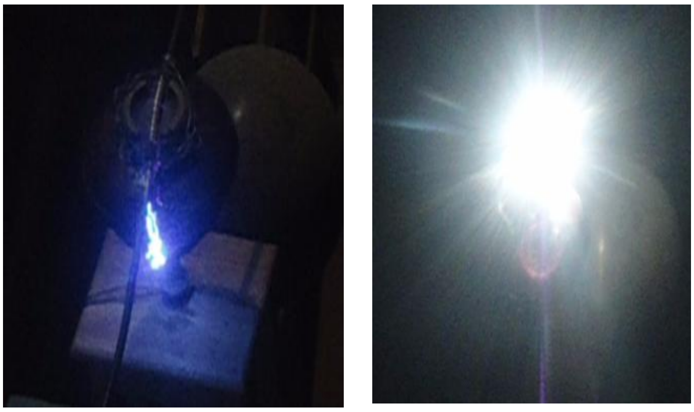

Fig. 20. Video Still Image of Flash over

5. CONCLUSIONS

A number of computer program packages have been developed for solution of finite element method. In this paper Comsol/M was used to plot the voltage stress over a porcelain insulator and to predict the power frequency breakdown voltages, and it is found that the simulation and experiment results are in close agreement. The simulation predicted that breakdown starts at a static voltage value of about 105 kV peak, at the value of ionization field intensity (30 kV/cm in atmospheric air at standard conditions).

Experimentally, it was observed that partial discharge corona commenced at around 72 kV rms, [101 kV peak]. Thus, it may be concluded that the finite-element packages can be used with a fair degree of accuracy to design insulators. In Red Sea State in eastern Sudan there is a seriously problem in the high voltage grid caused by the contamination, especially the salt pollution on the insulator surface, the paper is pointed out for a further study to solve this problem.

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