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# Incremental Linear Programming Optimal Power Flow: Including the VAR Cost Function

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**Abstract**: In this paper, the full AC incremental linear programming optimal power flow using POWERWORLD® Simulator and Microsoft® Excel is presented. A proposed formulation of including the VAR cost function to the objective function and an implementation of the proposed formulation in a 6-bus test system and the IEEE 30 bus system was made in order to decide whether this formulation favorably or unfavorably affects the optimization process. Research proved that this formulation can improve the total optimization process but not for all system types and sizes; the effect was favorable for the 30-bus system and unfavorable for the 6-bus system.

**Keywords:** *AC optimization; Linear programming; Optimal power flow; Reactive power pricing; Optimization improvement.*

## 1. INTRODUCTION

The Optimal Power Flow (OPF) problem is large and complicated non-linear optimization problem. It's a combination between Economic Dispatch (ED) and Power Flow (PF) calculation in which by calculating the dependent and control variables of the objective function through the power flow calculation and solve the optimization problem as same as solved through ED. The objective of the OPF is to find an optimum secured system, optimum for minimizing total generation cost and total losses, secured for all operating parts that must run at their limits such as generators, bus-bars, transformers and transmission lines.

Several methods are used to solve this problem such as non-linear methods, linear methods…etc., as stated in [1] and [2]. The non-linear methods are suffering from some difficulties. Lambda iteration and Newton based methods have been found to converge very fast but have difficulties in handling the inequality constraints, the gradient method is suffering from both convergence speed and inequality constraints, but these drawbacks did not exist in Linear Programming (LP) methods such as the Incremental LP method [2], [3].

The full AC Optimal Power Flow iterative LP method or the incremental LP method as in [2] is formulated by "linearizing the nonlinear objective function and constraints of the OPF AC power flow formulation around the current operating point using a first order Taylor series expansion in order to create a convex LP problem", which is formulated in terms of the increments of the control variables. Paper [2] proved that this method possesses speed and flexibility during calculation and produces reliable results for all system types and sizes.

As observed, reactive power pricing is not included in the formulation of paper [2] and reference [3]. Reactive power plays an important role in real power transfer and effects power system operation in numerous ways [4,5]. Pricing of reactive power is very important for the deregulated electric industry both financially and operationally. Financially through improving the economic efficiency of the system, operationally, the system efficiency and reliability will be improved by the reduction of the total transmission losses and the improvement of the voltage profile of the network [6].

In this paper, the inclusion of reactive power cost function to the objective function formulated in [2] and [3] is introduced. In incremental LP method, reactive power is already optimized therefore the inclusion of reactive power to the objective function is for improving the optimization process. If the influence of this inclusion is favorable i.e. improving the optimization process for the real power, then it can be included. If the influence is unfavorable then the VAR cost function must not be included. Note that in [2] the trust region method was used, which is not included in this paper.

## 2. PROBLEM FORMULATION

A. The Optimal Power Flow Formulation combining the Economic dispatch and the Power Flow:

– The objective function:

$$min \sum_{i=1}^{n} F_i\left(P_{gen_i}\right), \text{ Same as ED} \quad (1)$$

– Subjected to the equality constraint:

$$\sum_{i=1}^{N} P_{gen_i} = P_{Totalload} + P_{Totallosses}, \text{ Same as ED} \quad (2)$$

– Subjected to the inequality constraints:

$$P_{gen_i}^{min} \le P_{gen_i} \le P_{gen_i}^{max}$$
$$Q_{gen_i}^{min} \le Q_{gen_i} \le Q_{gen_i}^{max}$$
$$P_{ij}^{min} \le P_{ij} \le P_{ij}^{max}$$
$$Or, S_{ij}^{min} \le S_{ij} \le S_{ij}^{max}$$
$$V_i^{min} \le V_i \le V_i^{max}, for\ i = 1,2,3,\ldots,n$$

Where $P_{\text{gen}_i}$, $Q_{\text{gen}_i}$, $V_i$, $P_{ij}$ and $S_{ij}$ are the real generated power at generator $i$, the reactive generated power at generator $i$, the voltage of bus $i$, the real power flow at line$ij$ and the complex or the apparent power flow at line $ij$ respectively. These variables are calculated through the power flow solution [3].

B. The Power Flow Equation:

$$\frac{P_{gen_i} - jQ_{gen_i}}{V_i^*} = V_i \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} V_j \quad (3)$$

$$\therefore P_{gen_i} - jQ_{gen_i} = V_i^* \left[V_i \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} V_j\right] \quad (4)$$

$$\therefore P_{gen_i} = \Re\left\{V_i^* \left[V_i \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} V_j\right]\right\} \quad (5)$$

$$And Q_{gen_i} = -\Im\left\{V_i^* \left[V_i \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} V_j\right]\right\} \quad (6)$$

$$P_{ij} = \Re\left\{V_i\left[\left(V_i - V_j\right)y_{ij} + V_i^2 y_{shunt_{ij}}\right]^*\right\} \quad (7)$$

$$S_{ij} = abs\ \left\{V_i\left[\left(V_i - V_j\right)y_{ij} + V_i^2 y_{shunt_{ij}}\right]^*\right\} \quad (8)$$

Where:

$y_{ij} \equiv$ *the ij termof the admittance matrix*
$V_i^* \equiv$ *the conjugate value of the complex voltage at bus i*
$y_{shunt_{ij}}$
$\equiv$ *the shunt charging admittance to ground of lineij*

Therefore, the OPF equality constraint is written as:

– The equality constraint:

$$\left(P_{gen_i} - P_{load_i}\right) - j\left(Q_{gen_i} - Q_{load_i}\right) = V_i^* \left[V_i \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} V_j\right] \quad (9)$$

$$P_{gen_i} - P_{load_i} = \Re\left\{V_i^* \left[V_i \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} V_j\right]\right\} \quad (10)$$

$$Q_{gen_i} - Q_{load_i} = -\Im\left\{V_i^* \left[V_i \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i\neq j}}^{n} y_{ij} V_j\right]\right\} \quad (11)$$

C. Incremental LP Method:

In the full AC power flow using Newton-Raphson method [7], the following problem is solved:

$$[\mathcal{J}]\begin{bmatrix}\Delta\delta \\ \Delta V\end{bmatrix} = \begin{bmatrix}\Delta P_{gen} \\ \Delta Q_{gen}\end{bmatrix} \quad (12)$$

Where:
$\mathcal{J} \equiv$The Jacobean matrix.
$\Delta P \& \Delta Q \equiv$ are the change in power due to the change of voltage magnitudes $\Delta V$ and their phase angles $\Delta\delta$.

In the Incremental LP method and since using first order Taylor series expansion, the optimization process will be written in terms of $\Delta P_{\text{gen}}$, $\Delta Q_{\text{gen}}$, $\Delta V$ and $\Delta\delta$ where:

$$F_i\left(P_{gen_i}\right) = F_i\left(P_{gen_i}\right) + F_i\left(P_{gen_i}\right)'\left(P_{scheduled_i} - P_{gen_i}(V,\delta)\right) \quad (13)$$

The LP OPF should be started by a base power flow solution; here the power flow solution is designated as power flow zero (PF0) and the values of the base power flow solution are designated as:

$$P_{gen}^0, Q_{gen}^0, V^0\ and\ \delta^0$$

The linearized objective function of the incremental LPOPF is:

$$min \sum_{i=1}^{n} \left[F_i\left(P_{gen_i}^0\right) + \frac{dF_i\left(P_{gen_i}{}^0\right)}{dP_{gen_i}{}^0}\Delta P_{gen_i}\right] \quad (14)$$

Where:

$F_i\left(P_{\text{gen}_i}^0\right) \equiv$The objective function in terms of the base PF solution values.
$\frac{dF_i(P_{\text{gen}_i}{}^0)}{dP_{\text{gen}_i}{}^0} \equiv$The incremental cost function in terms of the base PF solution.

$\because F_i\left(P_{\text{gen}_i}^0\right)$ is considered to be as constant, it can be eliminated from the objective function, therefore the linearized objective function becomes:

$$min \sum_{i=1}^{n} \left[\frac{dF_i(P_{gen_i}{}^0)}{dP_{gen_i}{}^0}\Delta P_{gen_i}\right] \quad (15)$$

In order to linearize the real and reactive power equality constraints, the constraints of the power flow solution are formulated similar to the expression of the N-R method except that all the variables are included even the slack bus variables, and there is no need for the inversion of the Jacobean matrix to calculate $\Delta\delta_i$ and $\Delta V_i$ since the LP optimization is responsible of calculating these values [3]. The linearized real and reactive power equality constraints are:

$$\begin{bmatrix}\begin{bmatrix}\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_n}{\partial \delta_1}\\ \vdots & \ddots & \vdots\\ \frac{\partial P_1}{\partial \delta_n} & \cdots & \frac{\partial P_n}{\partial \delta_n}\end{bmatrix} & \begin{bmatrix}\frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_n}{\partial V_1}\\ \vdots & \ddots & \vdots\\ \frac{\partial P_1}{\partial V_n} & \cdots & \frac{\partial P_n}{\partial V_n}\end{bmatrix}\\ \begin{bmatrix}\frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_n}{\partial \delta_1}\\ \vdots & \ddots & \vdots\\ \frac{\partial Q_1}{\partial \delta_n} & \cdots & \frac{\partial Q_n}{\partial \delta_n}\end{bmatrix} & \begin{bmatrix}\frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_n}{\partial V_1}\\ \vdots & \ddots & \vdots\\ \frac{\partial Q_1}{\partial V_n} & \cdots & \frac{\partial Q_n}{\partial V_n}\end{bmatrix}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_n\\ \Delta V_1\\ \vdots\\ \Delta V_n\end{bmatrix} = \begin{bmatrix}P_{scheduled_1} - P_1(V,\delta)\\ \vdots\\ P_{scheduled_n} - P_n(V,\delta)\\ Q_{scheduled_1} - Q_1(V,\delta)\\ \vdots\\ Q_{scheduled_n} - Q_n(V,\delta)\end{bmatrix} \quad (16)$$

Where $\Delta\delta_1$, $\Delta V_1, \Delta P_{\text{load}_i}$ and $\Delta Q_{\text{load}_i}$ are taken as constants and equal to zero.

The inequality constraints are formulated as:

- The generator real power limits:

$$\Delta P_{gen_i} \geq \left(P_{gen_i}{}^{min} - P_{gen_i}{}^{0}\right)(\forall\ generators\, i)$$

$$\Delta P_{gen_i} \leq (P_{gen_i}{}^{max} - P_{gen_i}{}^{0})\ \ (\forall\ generators\, i)$$

- The generator reactive power limits:

$$\Delta Q_{gen_i} \geq (Q_{gen_i}{}^{min} - Q_{gen_i}{}^{0}\ )(\forall\ generators\, i)$$

$$\Delta Q_{gen_i} \leq (Q_{gen_i}{}^{max} - Q_{gen_i}{}^{0}\ )\ (\forall\ generators\, i)$$

- The bus voltage magnitude limits:

$$\left(V_i{}^{min} - V_i{}^{0}\right) \leq \Delta V_i \leq (V_i{}^{max} - V_i{}^{0}\ )\ (\forall\ buses\, i)$$

- The phase angle limits:

$$\left(\delta_i{}^{\min} - \delta_i{}^{0}\right) \leq \Delta\delta_i \geq (\delta_i{}^{\max} - \delta_i{}^{0}\ )(\forall \text{ buses } i)$$

D. The Full ACOPF Incremental LP method General Formulation:

$$\min \sum_{i=1}^{n}\left[\frac{\mathrm{d}F_i(P_{\text{gen}_i}{}^{0})}{\mathrm{d}P_{\text{gen}_i}{}^{0}}\Delta P_{\text{gen}_i}\right]$$

Subject to:

$$\sum_{i=1}^{n}\frac{\partial P_i(V,\delta)}{\partial V_i}\Delta|V_i| + \sum_{i=1}^{n}\frac{\partial P_i(V,\delta)}{\partial \delta_i}\Delta|\delta_i| + \sum_{i=1}^{n}\frac{\partial P_i}{\partial t_{ij}}\Delta t_{ij} = \Delta P_{\text{gen}_i}$$

$$\sum_{i=1}^{n}\frac{\partial Q_i(V,\delta)}{\partial V_i}\Delta|V_i| + \sum_{i=1}^{n}\frac{\partial Q_i(V,\delta)}{\partial \delta_i}\Delta|\delta_i| + \sum_{i=1}^{n}\frac{\partial Q_i}{\partial t_{ij}}\Delta t_{ij} = \Delta Q_{\text{gen}_i}$$

$$\sum_{i=1}^{N_{\text{gen}}} P_{\text{gen}_i}{}^{0} + \Delta P_{\text{gen}_i} = \sum_{i=1}^{N_{\text{gen}}} P_{\text{gen}_i} + P_{\text{loss}}$$

$$\sum_{i=1}^{N_{\text{gen}}} \mathrm{Q}_{\text{gen}_i}{}^{0} + \Delta\mathrm{Q}_{\text{gen}_i} = \sum_{i=1}^{N_{\text{gen}}} \mathrm{Q}_{\text{gen}_i} + \mathrm{Q}_{\text{loss}}$$

$$\Delta P_{\text{gen}_i} \geq \left(P_{\text{gen}_i}{}^{\min} - P_{\text{gen}_i}{}^{0}\right)(\forall \text{ generators } i)$$

$$\Delta P_{\text{gen}_i} \leq (P_{\text{gen}_i}{}^{\max} - P_{\text{gen}_i}{}^{0}\ )\ (\forall \text{ generators } i)$$

$$\Delta Q_{\text{gen}_i} \geq \left(Q_{\text{gen}_i}{}^{\min} - Q_{\text{gen}_i}{}^{0}\right)(\forall \text{ generators } i)$$

$$\Delta Q_{\text{gen}_i} \leq (Q_{\text{gen}_i}{}^{\max} - Q_{\text{gen}_i}{}^{0}\ )\ (\forall \text{ generators } i)$$

$$\left(V_i{}^{\min} - V_i{}^{0}\right) \leq \Delta V_i \leq (V_i{}^{\max} - V_i{}^{0}\ )\ (\forall \text{ buses } i)$$

$$(\delta_i{}^{\min} - \delta_i{}^{0}) \leq \Delta\delta_i \leq (\delta_i{}^{\max} - \delta_i{}^{0})\ (\forall \text{ buses } i)$$

$$\left(t_{ij}{}^{\min} - t_{ij}{}^{0}\right) \leq \Delta t_{ij} \leq \left(t_{ij}{}^{\max} - t_{ij}{}^{0}\right)(\forall \text{ transformer } ij)$$

$$\Delta V_{i_{\text{ref.}}}, \Delta\delta_{i_{\text{ref.}}}, \Delta P_{\text{load}_i} \text{and } \Delta Q_{\text{load}_i} = 0$$

Where$t_{ij}$ is transformer tap ratio in case of a transformer between bus i and j.

## 3. THE INCLUSION OF REACTIVE POWER COST FUNCTION TO THE OBJECTIVE FUNCTION

Many approaches of reactive power cost allocation are introduced such as in [8-11]. Based on [12], the conventional reactive power operating cost function is used since minimization of total operating cost is only considered:

$$\mathrm{CostQ}_i = \text{profit rate} * \mathrm{b} * \mathrm{Q}_i^2 \quad (17)$$

Where b $\equiv$ the cost coefficient of the input-output cost curve.

Profit rate$\equiv$ the profit rate of the real power and usually ranged from 5% to 10%.In this paper the profit rate is taken as 5% or 0.05.This equation only considers the operating cost of reactive power [8]. Linearizing equation (17) using first order Taylor series expansion (Eq. 13):

$$\mathrm{F_i}\left(\mathrm{Q_{gen_i}}\right) + \frac{\mathrm{dF_i}\left(\mathrm{Q_{gen_i}}\right)}{\mathrm{dQ_{gen_i}}}\Delta\mathrm{Q_{gen_i}} \quad (18)$$

Then the objective function of the incremental LPOPF becomes:

$$\min \sum_{i=1}^{n}\left[\frac{\mathrm{d}F_i(P_{\text{gen}_i}{}^{0})}{\mathrm{d}P_{\text{gen}_i}{}^{0}}\Delta P_{\text{gen}_i} + \frac{\mathrm{d}F_i(Q_{\text{gen}_i}{}^{0})}{\mathrm{d}Q_{\text{gen}_i}{}^{0}}\Delta Q_{\text{gen}_i}\right] \quad (19)$$

Subject to:

$$\sum_{i=1}^{n}\frac{\partial P_i(V,\delta)}{\partial V_i}\Delta|V_i| + \sum_{i=1}^{n}\frac{\partial P_i(V,\delta)}{\partial \delta_i}\Delta|\delta_i| + \sum_{i=1}^{n}\frac{\partial P_i}{\partial t_{ij}}\Delta t_{ij} = \Delta P_{\text{gen}_i}$$

$$\sum_{i=1}^{n}\frac{\partial Q_i(V,\delta)}{\partial V_i}\Delta|V_i| + \sum_{i=1}^{n}\frac{\partial Q_i(V,\delta)}{\partial \delta_i}\Delta|\delta_i| + \sum_{i=1}^{n}\frac{\partial Q_i}{\partial t_{ij}}\Delta t_{ij} = \Delta Q_{\text{gen}_i}$$

$$\sum_{i=1}^{N_{\text{gen}}} P_{\text{gen}_i}{}^{0} + \Delta P_{\text{gen}_i} = \sum_{i=1}^{N_{\text{gen}}} P_{\text{gen}_i} + P_{\text{loss}}$$

$$\sum_{i=1}^{N_{\text{gen}}} \mathrm{Q}_{\text{gen}_i}{}^{0} + \Delta \mathrm{Q}_{\text{gen}_i} = \sum_{i=1}^{N_{\text{gen}}} \mathrm{Q}_{\text{gen}_i} + \mathrm{Q}_{\text{loss}}`$$

$$\Delta P_{\text{gen}_i} \geq \left(P_{\text{gen}_i}{}^{\min} - P_{\text{gen}_i}{}^{0}\right)(\forall \text{ generators } i)$$

$$\Delta P_{\text{gen}_i} \leq (P_{\text{gen}_i}{}^{\max} - P_{\text{gen}_i}{}^{0}) \ (\forall \text{ generators } i)$$

$$\Delta Q_{\text{gen}_i} \geq \left(Q_{\text{gen}_i}{}^{\min} - Q_{\text{gen}_i}{}^{0}\right)(\forall \text{ generators } i)$$

$$\Delta Q_{\text{gen}_i} \leq (Q_{\text{gen}_i}{}^{\max} - Q_{\text{gen}_i}{}^{0}) \ (\forall \text{ generators } i)$$

$$\left(V_i{}^{\min} - V_i{}^{0}\right) \leq \Delta V_i \leq (V_i{}^{\max} - V_i{}^{0}) \ (\forall \text{ buses } i)$$

$$(\delta_i{}^{\min} - \delta_i{}^{0}) \leq \Delta\delta_i \leq (\delta_i{}^{\max} - \delta_i{}^{0}) \ (\forall \text{ buses } i)$$

$$\left(t_{ij}{}^{\min} - t_{ij}{}^{0}\right) \leq \Delta t_{ij} \leq \left(t_{ij}{}^{\max} - t_{ij}{}^{0}\right)(\forall \text{ transformer } ij)$$

$$\Delta V_{i_{\text{ref.}}}, \Delta\delta_{i_{\text{ref.}}}, \Delta P_{\text{load}_i} \text{and } \Delta Q_{\text{load}_i} = 0$$

Figure 1 shows the solution algorithm for the Incremental LP OPF described in the following flowchart:

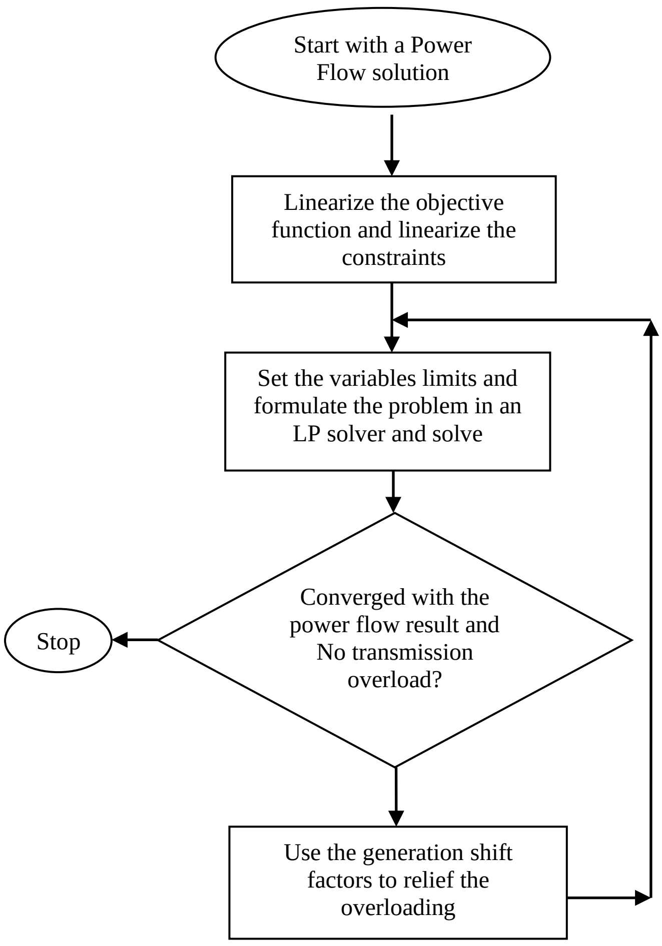


**Fig. 1.** Solution Algorithm for the Incremental LP OPF

## 4. SIMULATION AND RESULTS

In order to make the comparison between the incremental LP performance before the addition of the VAR cost function (described in section 2-D) and after the addition (the proposed formulation of section 3) in different optimization aspects such as minimization of total operating cost, minimization of total losses and system security improvement, an implementation of both formulations was made following section 3 solution algorithm in two test systems in order to make a decision about the influence of this formulation in the optimization process whether it is favorable or not in all system types and sizes. The first system is a 6-bus test system (data are available in [3]) and the second system is IEEE 30 bus test system (data are available in [7]).

### I. Implementation on the 6-bus test system:

- Initial power flow results PF0as shown in Table 1:

**Table 1.** Initial power flow results

| Bus No. | Generation MW | Generation MVAR | Bus PU Volt |
|---|---|---|---|
| 1 | 212.96 | -10.76 | 1.07 |
| 2 | 50 | 21.76 | 1.05 |
| 3 | 50 | 19.02 | 1.05 |
| 4 | 0 | 0 | 1.02721 |
| 5 | 0 | 0 | 1.02212 |
| 6 | 0 | 0 | 1.02458 |
| Total Gen | 312.96 | 30.02 | |
| Total losses | 12.96 | -14.98 | |
| Operating cost | 4478.91 $/hr. | 274.74 $/hr. | |
| Total Cost | 4753.65 $/hr. | | |

- Incremental LPOPF results before and after adding the VAR cost function PF0as shown in Tables2 and 3.

**Table 2.** LPOPF results before addition

| Bus No. | Generation MW | Generation MVAR | Bus PU Volt | Angles Radians |
|---|---|---|---|---|
| 1 | 110.01 | 7.18 | 1.07 | 0 |
| 2 | 125.83 | -10.8 | 1.05732 | -0.03 |
| 3 | 71.78 | 15.81 | 1.05982 | -0.06 |
| 4 | 0 | 0 | 1.02962 | -0.09 |
| 5 | 0 | 0 | 1.02867 | -0.11 |
| 6 | 0 | 0 | 1.03377 | -0.11 |
| Total Gen | 307.62 | 12.19 | | |
| Total losses | 7.62 | -32.81 | | |
| Operating cost | 4258$/hr | 159.9 $/hr | | |
| Total Cost | 4417.92$/hr | | | |

**Table 3.**LPOPF results after addition

| Bus No. | Generation MW | Generation MVAR | Bus PU Volt | Angles Radians |
|---|---|---|---|---|
| 1 | 115 | 22.26 | 1.07 | 0 |
| 2 | 121.21 | -20.18 | 1.04329 | -0.03 |
| 3 | 71.73 | 12.42 | 1.0441 | -0.06 |
| 4 | 0 | 0 | 1.01973 | -0.09 |
| 5 | 0 | 0 | 1.0171 | -0.11 |
| 6 | 0 | 0 | 1.01883 | -0.11 |
| **Total Gen** | **307.94** | **14.5** | | |
| **Total losses** | **7.94** | **-30.5** | | |
| **Operating cost** | **4263.8 $/hr.** | **362.156 $/hr.** | | |
| **Total Cost** | **4625.936 $/hr.** | | | |

– Reduction of total operating cost during each iteration before and after as shown in **Fig. 2**:

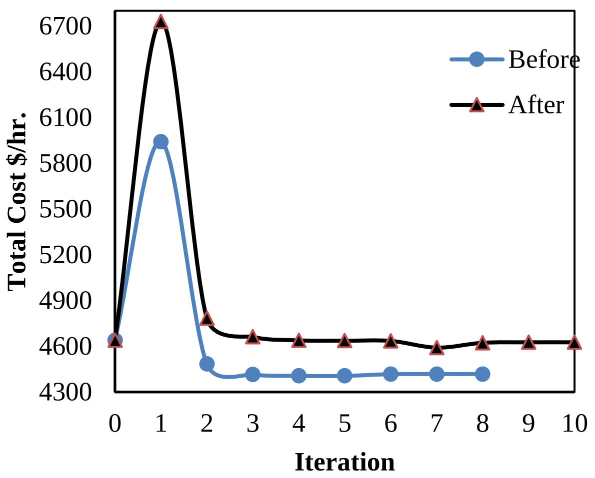


**Fig. 2.** Total cost reduction

– Reduction of total losses during each iteration before and after as shown in **Fig. 3** and **4**:

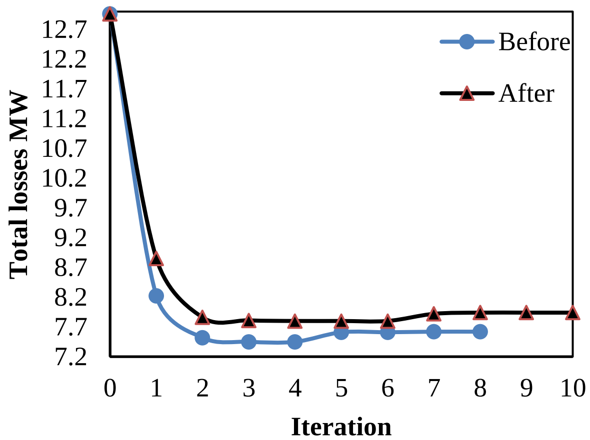


**Fig. 3.** Total loss reduction MW

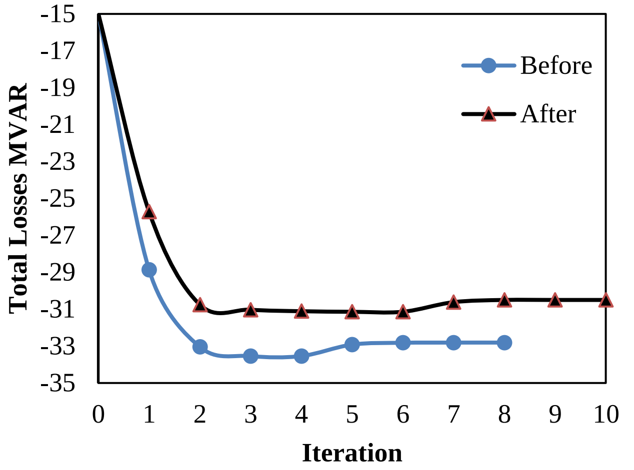


**Fig. 4.** Total loss reduction MVAR

– Voltage Profile before and after adding the VAR cost function as shown in **Fig. 5**:

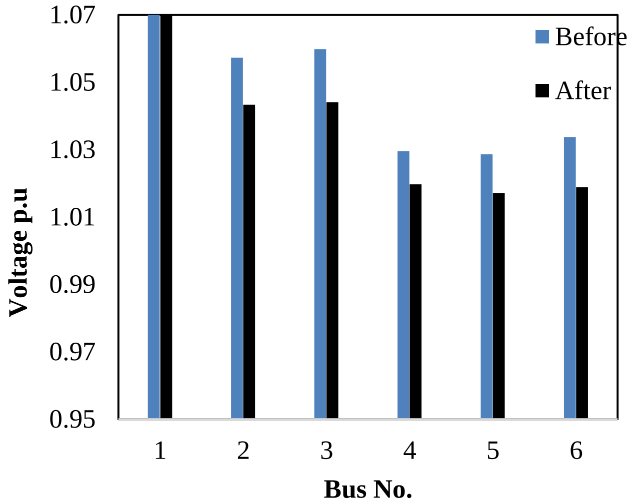


**Fig. 5.**Voltage Profile

Here the inclusion of the VAR cost function did not improve the optimization process where total operating cost and total losses are increased compared the case before the inclusion except that voltage profile is improved. However, in this system, the VAR cost function must not be added due to the unfavorable effect on the total optimization process while it can be used for pricing purposes only.

## II. Implementation on the IEEE 30 bus test system:

– Initial power flow results PF0PF0as shown in Table 4:

**Table 4.**Initial Power flow results

| Bus No. | Generation MW | Generation MVAR |
|---|---|---|
| 1 | 260.95 | -16.53 |
| 2 | 40 | 49.56 |
| 3 | 0 | 36.94 |
| 4 | 0 | 37.22 |
| 5 | 0 | 16.18 |
| 6 | 0 | 10.63 |
| **Total Generation** | **300.95** | **134** |
| **Total Load** | **283.4** | **126.2** |
| **Total Losses** | **17.55** | **7.8** |
| **Total Cost** | **875.256 $/hr.** | **591.8 $/hr.** |

– Incremental LPOPF results before and after adding the VAR cost functionPF0as shown in Tables 5 and 6**:**

**Table 5:** LPOPF results before addition

| Bus No. | Generation MW | Generation MVAR |
|---|---|---|
| 1 | 147.78 | 7.8 |
| 2 | 80 | -3.83 |
| 3 | 24.86 | 30.21 |
| 4 | 13.82 | 38.97 |
| 5 | 10.27 | 16.03 |
| 6 | 15.26 | 10.83 |
| **Total Generation** | **291.99** | **100.01** |
| **Total Load** | **283.4** | **126.2** |
| **Total Losses** | **8.59** | **-26.19** |
| **Real power cost** | **824.497 $/hr.** | |
| **Reactive power cost** | **355.951 $/hr.** | |
| **Total operating cost** | **1180.448 $/hr.** | |

**Table 6.**LPOPF results after addition

| Bus No. | Generation MW | Generation MVAR |
|---|---|---|
| 1 | 149.83 | 11.34 |
| 2 | 80 | 3.03 |
| 3 | 24.67 | 30.28 |
| 4 | 15.64 | 26.65 |
| 5 | 10 | 18 |
| 6 | 12 | 12.32 |
| Total Generation | 292.14 | 101.62 |
| Total Load | 283.4 | 126.2 |
| Total Losses | 8.74 | -24.58 |
| **Real power cost** | **823.515 $/hr.** | |
| **Reactive power cost** | **246.235 $/hr.** | |
| **Total operating cost** | **1069.75 $/hr.** | |

– **Reduction of total operating cost during each iteration before and after as shown in fig. 6:**

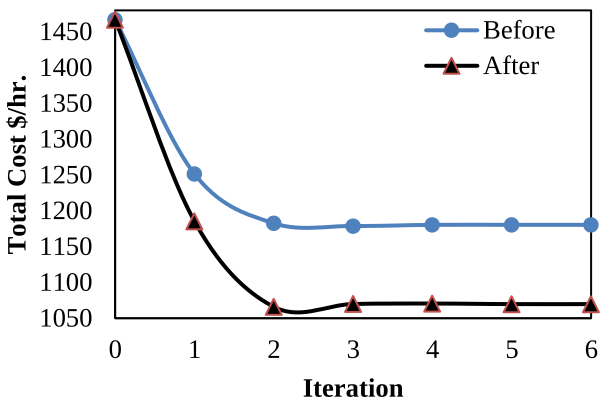


**Fig. 6.**Total cost reduction

– **Losses during each iteration before and after as shown in fig. 7 and 8:**

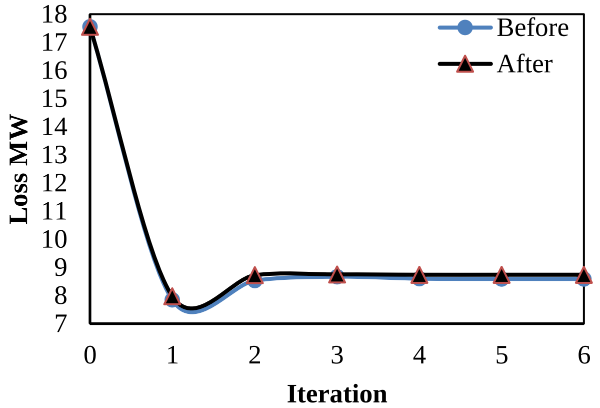


**Fig. 7.**Total loss reduction MW

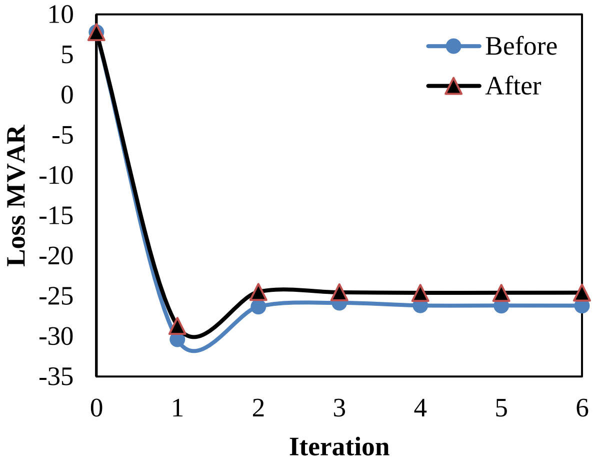


**Fig. 8.**Total loss reduction MVAR

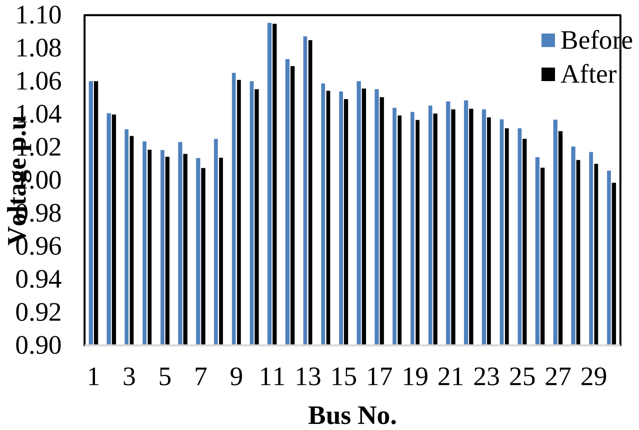


**Fig. 9.**Voltage profile

– Voltage Profile before and after adding the VAR cost functions shown in Fig. 9**:**

Here and unlike the 6-bus system, the inclusion of the VAR cost function improves the optimization process in total operating cost and voltage magnitudes which are reduced by a considerable amount than before the inclusion. Despite that before the inclusion has an advantage on total loss reduction, the addition of the VAR cost function in this system favorably affected the total optimization process and therefore it must be included.

## 5. CONCLUSIONS

Incremental LP method is very reliable, fast and flexible and it can be used in order to solve the OPF problem of any system types and sizes. In this paper and by the use of POWERWORLD® Simulator and Microsoft® Excel, a proposed formulation by adding the VAR cost function to the Incremental LPOPF function was presented. An implementation of the proposed formulation was made in a 6-bus system and the IEEE 30 bus system. The aim was to prove that this addition can improve the optimization process or not. Research proved that this formulation can improve the total optimization process but not at all system types and sizes as observed in the last section. Therefore, this inclusion affects the optimization process favorably and unfavorably, but as observed in the IEEE 30 bus system, a significant effect by considerable saving of total cost and improvement in voltage profile was presented. Therefore, if a trial is made using this formulation and favorably affect the optimization process it will be a benefit.

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