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On the Establishment of a Precise GPS Network in Khartoum State

Mudathir A. Mohammed, Rashad A. Khalil, Awadelgeed M. Awadelgeed, & Gamal H. Seedahmed

Dept. of Surveying Engineering Faculty of Engineering University of Khartoum

(E-mail: mudathirrex@gmail.com, ra.shad0490@gmail.com, awadelgeedmohamed@yahoo.com, Gamal.Seedahmed@gmail.com)

Abstract: The use of the Global Positioning System (GPS) as a fast and efficient alternative in the geodetic network measurements has become a common practice worldwide. The advantages of using the GPS technique compared to the traditional surveying methods are based on accuracy, time and cost. In this study, the static-mode of GPS measurement technique has been utilized to establish a precise geodetic network in Khartoum State. The network was referenced to a single control station that is related to the International Terrestrial Reference Frame (ITRF 2005). A number of six points have been constructed and observed around Khartoum State. In particular, seven observation sessions were conducted that gave an over determined system. The least-squares method was used in the adjustment of the GPS network. Two methods were utilized for the adjustment, namely, the parametric and the condition equation methods. The most probable values of the coordinates of the newly-established points were computed from both methods and they were found to be identical. The average standard deviation of the X, Y, and Z coordinates are ±1.36 cm, ±1.12 cm, & ±0.69 cm respectively.

Keywords: GPS, Geodetic Networks, Adjustment.

1. INTRODUCTION

The Global Positioning System (GPS) is a satellite-based positioning system, created and operated by the United States Department of Defence. It provides a fast, accurate and reliable way of determining the positions of points, 24 hours a day, all around the world and under all weather conditions [1]. Satellite-based positioning has become the standard technique in establishing geodetic networks. It provides high accuracy and few limitations compared to the traditional positioning methods.

The preferred method of adjusting geodetic data is the method of least squares estimation. It gives the most probable values of observations provided that the observations are redundant and the errors are random. The recent advances in computer technology allow for the application of least squares estimations to GPS observations with ease. This makes it possible to combine many redundant observations and to obtain a more reliable solution which can be qualitatively assessed.

In fact, the national geodetic network stations of Sudan were observed and computed using conventional triangulation techniques [2]. The classical observables are subject to some estimable errors and unknown systematic errors. The former are accounted for in the network adjustment in the variance-covariance matrix of observations, the latter tend to propagate systematically through the network causing some distortions.

Thus, the problems of classical geodetic networks are of a twofold nature, one being the inherent susceptibility of the classical observation techniques to systematic errors, and two, the incompleteness of the mathematical models applied. The observed inconsistencies in the national triangulation network in Sudan makes it unsuitable for many civil works, such as dam's Construction, and geodetic applications such as national mapping. Today, each survey project in the country has its own independent Absolute base station that is not relatively connected to the other Projects [2].This paper reviews the establishment of a precise geodetic network in Khartoum State, using GPS technology and least squares estimation and it is organized as follows: Section 2 gives a brief literature review of the topics of interest to this paper, namely, the Global Positioning System, the International Terrestrial Reference System and the Method of Least Squares. Section 3 gives a brief description of the network, the observation method and the hardware and software employed. Section 4 reviews the mathematical model used in the adjustment of the network. Section 5 presents the results and discussions and Section 6 concludes the paper.

2. Literature Review

2.1. Global Positioning System

GPS is a Global Positioning System based on satellite technology. The fundamental technique of GPS is to measure the ranges between the receiver and a few simultaneously observed satellites. The positions of the satellites are forecasted and broadcasted along with the GPS signal to the user. Through several known positions (of the satellites) and the measured distances between the receiver and the satellites, the position of the receiver can be determined (Xu, 2003) [3]. The GPS provides positioning and timing 24 hours per day, anywhere in the world, and under any weather conditions [1].

The highest level of accuracy is achieved using carrier phase-shift, dual frequency receivers and the employment of static, relative positioning methods. Compared to single frequency receivers, the dual frequency receivers collect the needed data faster, observe longer baselines with greater accuracy and eliminate certain errors, such as ionospheric refraction. The static, relative positioning method yields the coordinates of points relative to another point, by observing them simultaneously, i.e. at the same time, from the same satellites and at the same epoch rate. Furthermore, the double differencing technique employed in the processing of the data eliminates satellite clock bias, receiver clock bias and much of the ionospheric and tropospheric refraction from the solution [4].

2.2. International Terrestrial Reference System

The conventional terrestrial reference system, established and maintained by the International Earth Rotation Service (IERS), and nearly exclusively used for today's scientific and practical purposes is the International Terrestrial Reference System (ITRS); its realization is the International Terrestrial Reference Frame (ITRF) [5]. The system is geocentric. Its Z-axis is defined by the IERS reference pole (IRP) and its X-axis lies in the IERS reference meridian (IRM). The ITRF is realized by a number of terrestrial sites where temporal effects (plate tectonics, tidal effects) are also taken into account.

The GPS terrestrial reference system is the World Geodetic System 1984 (WGS-84). The refined WGS-84 frame, introduced in 2002 shows insignificant systematic differences in the order of 1cm with respect to the ITRF2005. Hence, both frames are virtually identical (Holfmann-Wellenhof et al, 2008) [6].

2.3. The Method of Least Squares

The method of least squares provides the most rigorous and statistically thorough estimation of the unknown parameters. The least squares estimates are defined as those which minimize a specified quadratic form of the weighted residuals. Thus, the fundamental condition of the least squares method is:

$$v^T W v = a\ minimum \qquad (1)$$

Where v is the vector of residuals and W is the weight matrix for the observations.

Some of the properties of the least squares method is that it gives a unique solution to a given problem, it enables all observations to be simultaneously included in an adjustment, and each observation can be weighted according to its estimated precision. The method leads to an easy quantitative assessment of the quality, e.g. via the covariance matrix of the estimates and it is a general method that can be applied to any problem. Furthermore, the estimates computed using this method are unbiased, and their covariance matrix has a smaller trace (i.e. smaller sum of variances) than any other linear unbiased estimate (Cross, 1994) [7].

3. Methodology: Planning and Field Work

The network consists of an ITRF2005 reference station, denoted by A_0, located at the Ministry of Urban Planning, in Khartoum, two Dam Implementation Unit (DIU) monuments denoted by PU_{34} and PU_{36}, located in Omdurman and three new monuments NP_1, and NP_2 in Khartoum North and NP_3 in Khartoum.

The GPS receivers used were dual frequency receivers, provided by the Sudan Surveying Authority (SSA), two Leica GPS900 receivers and one Leica GPS1200 receiver.

The observation sessions were designed such that

- Each of the baselines connecting the six points to A_0 would be observed once.
- Each of the baselines forming the external shape of the network would be observed at least once.
- Each point is incorporated into at least two different baseline observations.
- All of the above conditions are met without the incorporation of trivial baseline observations.

The distribution of the points and the sessions, together with the non-trivial baselines are shown in Fig. (1).

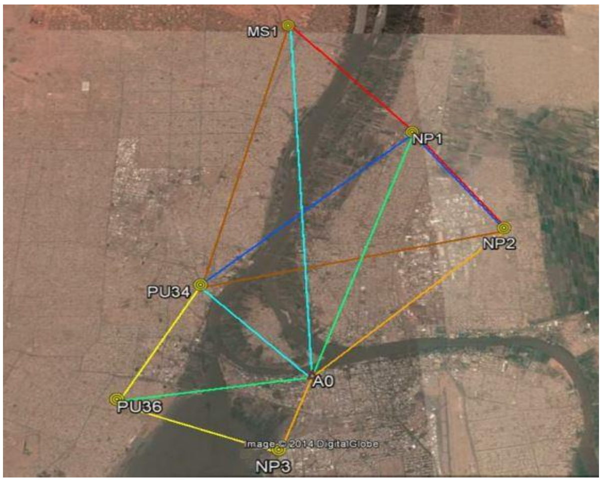

Fig (1): Points distribution.

The observation time for each session was calculated based on the longest baseline using the equation 20min + 2mins/km (Ghilani and Wolf, 2008) [4]. The atmospheric data for the observation date, together with satellite availability charts, skyplots and DOP charts were acquired from the Trimble website.

The software package, Leica Geo Office Combined v7, owned by the SSA was used in the post processing of the GPS data. The observations were processed using a single baseline solution and broadcast ephemeris. The outputs of the post-processing consisted of baseline components (ΔX, ΔY and ΔZ), the standard deviation of each component and the cofactor matrix elements for each baseline.

The analyses of the repeated baseline measurements and loop closures revealed inconsistencies in the observations of the seventh session compared to the other sessions, and as a result, the seventh session was repeated, and the new results were found to be consistent with the other sessions.

All the pre-adjustment analysis and adjustment computations were performed using MATLAB R2010a software. The network was adjusted using both, the Parametric (Observation) Equation model and the Condition Equation model. The ITRF2005 control point A_0 was held to be fixed in the calculations and its coordinates were assumed to be free from error.

4. Methodology: Mathematical Modeling

4.1. Parametric Method

For line ij, an observation equation can be written for each baseline component observed as

$$\begin{aligned} X_j &= X_i + \Delta X_{ij} + v_{X_{ij}} \\ Y_j &= Y_i + \Delta Y_{ij} + v_{Y_{ij}} \\ Z_j &= Z_i + \Delta Z_{ij} + v_{Z_{ij}} \end{aligned} \qquad (2)$$

In general, any group of observation equations may be represented in matrix form as

$$AX = L + V \qquad (3)$$

Where

A ... Design matrix
X ... Vector of unknowns
L ... Vector of observations
V ... Vector of residuals

By introducing in addition the definitions

σ_0^2 ... A priori variance

Σ_L ... Covariance matrix of observations

W ... Weight matrix of observations

$$W = \frac{1}{\sigma_0^2}\Sigma_L \quad (4)$$

The application of least squares principle on the observation equations above leads to the solution

$$X = (A^T W A)^{-1}(A^T W L) \quad (5)$$

The residuals vector after adjustment is

$$V = AX - L \quad (6)$$

The reference variance can be computed as

$$\hat{\sigma}_0^2 = \frac{V^T W V}{r} \quad (7)$$

The covariance matrix of the adjusted quantities is

$$\Sigma_X = \hat{\sigma}_0^2 (A^T W A)^{-1} \quad (8)$$

Then the individual adjusted quantities σ_{X_i} can be computed as

$$\sigma_{X_i} = \sqrt{\Sigma_{X_i X_i}} \quad (9)$$

Where

$\Sigma_{X_i X_i}$ ... Diagonal element (i, i) of Σ_X.

4.2. Condition Method

The condition equations model is

$$F(L_a) = 0 \quad (10)$$

Where

L_a ... Vector of n adjusted observations

F ... Vector function of r equations

r ... Degree of freedom

If L_b denotes the vector of observations, then the residuals are defined by

$$V = L_a - L_b \quad (11)$$

So the mathematical model can be written as

$$F(L_b + V) = 0 \quad (12)$$

By using the first-order of Taylor series expansion around the known point of expansion (L_b), giving

$$BV + E = 0 \quad (13)$$

With

$$B = \frac{\partial F}{\partial L_a}\Big|_{L_b} \quad (14)$$

$$E = F(L_b) \quad (15)$$

Then, the residuals can be obtained from:

$$V = -W^{-1}B^T(BW^{-1}B^T)^{-1}E \quad (16)$$

The adjusted observations vector follows from

$$L_a = L_b + V \quad 8 \quad (17)$$

The covariance matrix of the adjusted observations is

$$\Sigma_{L_a} = \hat{\sigma}_0^2 S W^{-1} S^T \quad (18)$$

With

$$\sigma_0^2 = \frac{V^T W V}{r} \quad (19)$$

$$S = I - W^{-1}B^T(BW^{-1}B^T)^{-1} \quad (20)$$

Where

I ... Identity matrix with n dimension

The adjusted coordinates of the point j can compute from

$$\hat{X}_j = X_i + \Delta\hat{X}_{ij}$$
$$Y_j = Y_i + \Delta\hat{Y}_{ij} \quad (21)$$
$$\hat{Z}_j = Z_i + \Delta\hat{Z}_{ij}$$

Where

X_i, Y_i & Z_i ... Coordinates of the point i

$\Delta\hat{X}_{ij}, \Delta\hat{Y}_{ij}$ & $\Delta\hat{Z}_{ij}$ ... Adjusted components of the baseline ij.

The covariance matrix of the point j can be computed from:

$$\Sigma_j = J\Sigma_{i\Delta}J^T \quad (22)$$

Where

J ... Jacobian matrix

$\Sigma_{i\Delta}$ ... Covariance matrix of the i point

Coordinates and the ij baseline components.

5. Results and Discussion

The most probable values of the coordinates for the six new control points were computed using the parametric and condition methods of least squares estimation, the results were found to be identical. The mean standard deviations of $x, y,$ & z are ±1.36 cm, ±1.12 cm, & ±0.69 cm respectively (see Table 1). Table 2 shows the coordinates of the six points and their accuracy.

Table 1: Overall accuracy of the six points.

Standard deviation	Min (cm)	Max (cm)	Mean (cm)
σx	1.16	1.82	1.36
σy	0.97	1.35	1.12
σz	0.64	0.85	0.69

The station NP_3 has the lowest accuracy, in both horizontal and vertical components; this is attributed to low redundancy as the station was only occupied twice (see Fig. 2). The highest accuracies were achieved in stations PU_{34} and NP_1; these are ascribed to high redundancy in the case of PU_{34} which was occupied four times and the complete lack of obstruction around NP_1

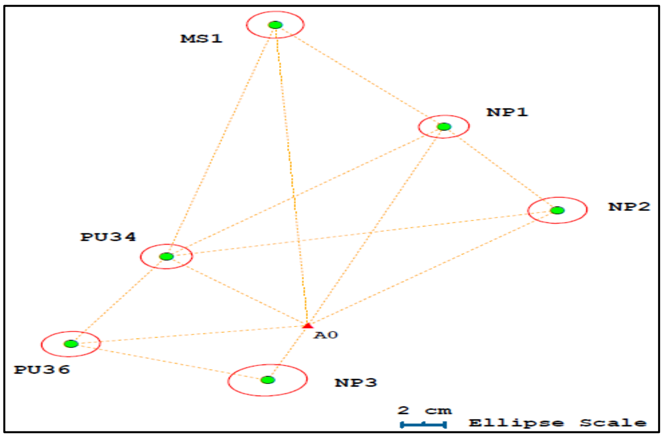

Fig (2): Network configuration.

Table 2: Coordinates of the adjusted points.

Station	X (m)	Y (m)	Z (m)	σx (m)	σy (m)	σz (m)
MS1	5178944.7038	3300748.7160	1717375.0713	0.0131	0.0115	0.0071
PU34	5182536.4811	3300088.3405	1707821.1833	0.0119	0.0104	0.0064
PU36	5184567.1029	3298774.9710	1704213.1367	0.0135	0.0108	0.0067
NP1	5177830.9228	3304666.4067	1713189.0183	0.0116	0.0097	0.0065
NP2	5177241.1116	3307403.8565	1709744.7172	0.0136	0.0112	0.0068
NP3	5182471.8686	3302830.2985	1702726.1106	0.0182	0.0135	0.0085

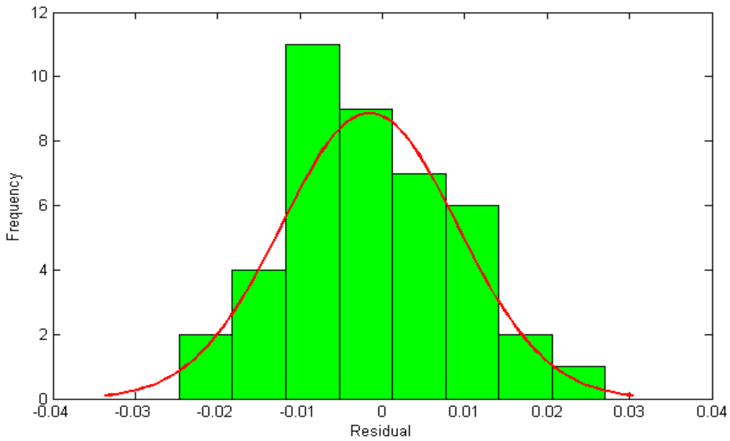

Fig. (3): Residuals distribution.

6. Conclusion

This paper presented a GPS network that was established from a single control station (ITRF 2005), using static relative positioning technique and single baseline processing. The network was established using redundant observations which formed an over determined system. The most probable values of the coordinates were estimated using the parametric and condition methods of least squares; the result were found to be identical. The accuracies were assessed and the reliability of the network was determined where the mean standard deviations of $x, y, \& z$ are ±1.36 cm, ±1.12 cm & ±0.69 cm respectively.

The accuracy of the network can be improved by incorporating more than one reference station, using Global Navigation Satellite Systems (GNSS) receivers, increasing the observation session time and further improvement can be achieved by using precise ephemerides rather than broadcast ephemerides.

The formal standard errors of the vector computations given by the software are generally optimistic by a factor of three to ten times [8]. Therefore, they don't give a right estimate of the positioning error, so they should never be used as an indicator of accuracy. The correct estimates of the errors can be computed after adjusting the network using the method of least squares.

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