



Simulation of River Bed Changes Upstream Merowe Dam

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Abstract: The main objective of this paper is to address long term morphological changes for Merowe Dam Reservoir located in Sudan using River Analysis System Software (HEC-RAS 5.0.3) . The trend of three different sediment transport equations (Laursen/Copeland, Yang and Ackre/White) had been represented in terms of the volumetric bed changes. The results obtained by the model had been calibrated and validated, furthermore, Sensitivity analysis was conducted. For the model, Two different boundary conditions had been used, one is the Sediment rating curve and the other is equilibrium load. Using the predictive model concept, three bed changes scenarios were simulated, assuming repeated flows, assuming wet and dry flows, Model limitations were considered and further recommendations were highlighted.

Keywords: *Morphological Change, Merowe Dam, Reservoir Sedimentation, Sensitivity Analysis, HEC-RAS.*

1. INTRODUCTION:

1.1 STUDY AREA:

Merowe Dam is located in the Northern State of Sudan at zone 36 N UTM (longitude 32.0532° and Latitude 18.6689°) at the main stream of River Nile, the dam is approximately 350 Km north of the capital Khartoum, the site location is slightly down stream of the Fourth Cataract of the River Nile as shown in fig (1), the dam operation started in 2009. (L.D.SCHEWE, 2006).

1.2 PROBLEM STATEMENT

Merowe Dam reservoir has a volume of 12.45 billion m³ at full supply level of 300.00 a.m.s.L. It worth mentioning that River Nile Basin has an area of 2.87 million Km² and a mean annual discharge of 2,514 m³/s (L.D.SCHEWE, 2006). The large volume of Merowe dam reservoir acts as a huge sediment trap and disturbs natural equilibrium of the stream, so that when water velocity decreased, large amount of sediment is deposited within the reservoir basin causing significant changes in river-bed profiles.



Fig. 1: MEROWE DAM LOCATION (GOOGLE, n.d.)

1.3 OBJECTIVES:

The objectives of this research are:

- i) Long-term simulation for river bed changes at Merowe Dam reservoir for different hydrological conditions.
- ii) To quantify total sediment load using different sediment transport equations (Laursen/Copeland, Yang and Ackre/White).
- iii) To compare the differences in total sediment load when applying different sediment transport equations.
- iv) To investigate capability of quasi-unsteady flow / sediment modeling to route sediment movement in the reservoir considering dam operation

.HEC-RAS 5.0.1 software was used for the simulation of morphological changes and calculations of sediment transport capacity for relatively long term as a result of erosion and deposition in a stream channel, quasi-unsteady flow model coupled with sediment model was used for this purpose.

2. METHODOLOGY

2.1 HEC RAS MODEL

The general concept for simulating morphological river-bed changes using HEC-RAS Model is presented in fig (2). This model was built for the evaluation of sediment transport for relatively long term to simulate the erosion and deposition in Merowe reservoir. Quasi-unsteady flow model coupled with sediment model was used for this purpose.

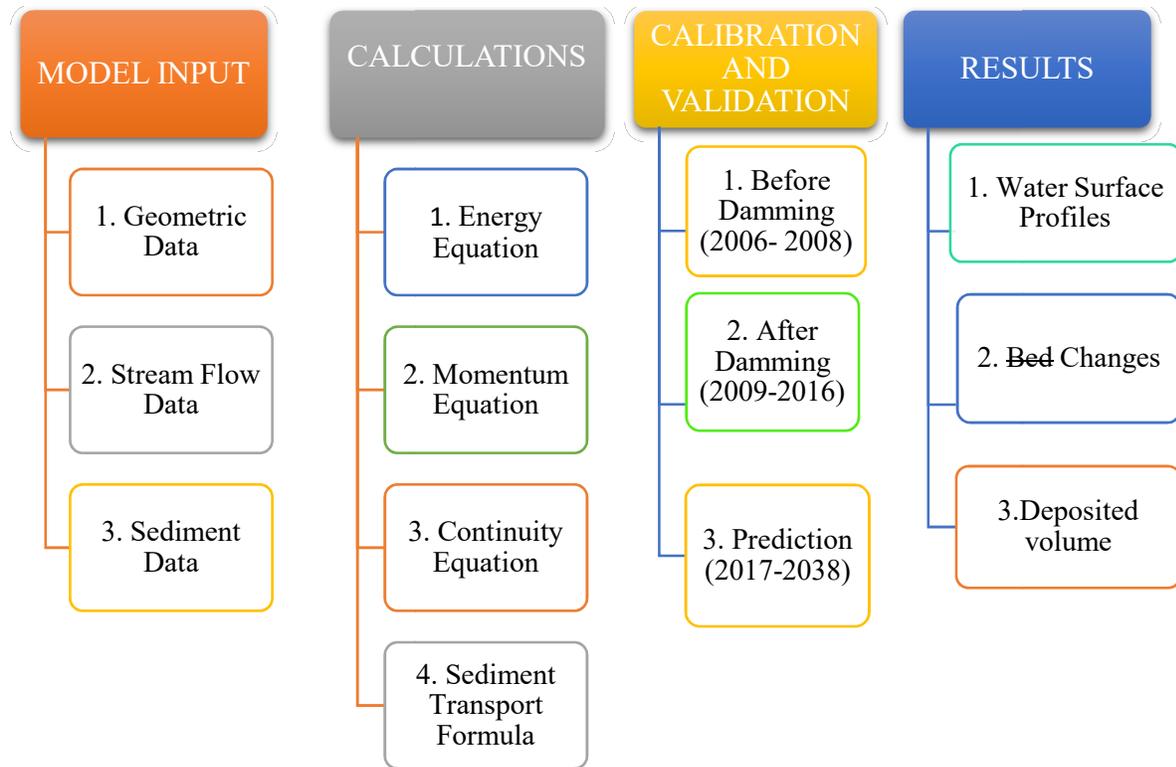


Fig. 2: GENERAL CONCEPT OF HEC-RAS SEDIMENT MODELING

2.2 MODEL CONSTRUCTION:

A reach with a length of 219 km was used to build the model. Geometric data were developed by extracting cross section from (X, Y,Z) data provided by bathymetric survey carried out in (2004) for a total of 107 cross sections with mean distance of 2 km between each as presented in fig (3). The main channel course was identified using historical image from GOOGLE EARTH.

The dam was represented in the model using inline structure module, three groups of gates were defined, the first group consists of 12 gate-openings representing bottom outlets, the second group consists of 2-gate openings representing the two surface overflow spillways, and the last one is of 6 deep sluices, for each group the geometric data were set, the type of gates was radial and the spillway was

selected as an ogee shape. The weir width of 10 m and distance of 45 m between the upstream of the weir and the downstream cross-section was defined, dam crest level at 304 m was entered to define the top of the dam.

2.3 HYDRODYNAMIC MODEL:

2.3.1 STEADY FLOW MODEL:

For the steady flow model, boundary conditions were defined at the most upstream boundary (ELKURU station) and the most downstream boundary (ELHESSAI- station) to establish the starting water surface at the ends of the river system, the boundary conditions used in the steady flow model were the rating curves at both (ELKURU) and (ELHESSAI), the location of the two stations is presented in figure (3).



Fig. 3: GOOGLE EARTH KMZ FOR CROSS SECTIONS LOCATION OF UPSTREAM AND DOWNSTREAM BOUNDARY CONDITIONS

2.3.2 MODEL CONSIDERATIONS:

- 1) Flow is steady, water surface profiles are calculated from cross-section to another by solving the Energy equation using standard step method iterations.
- 2) Flow is gradually varied, whenever flow is rapidly varied, the momentum equation will be applied.
- 3) River Channels should have small slopes (less than 1:10).

2.3.3 QUASI UNSTEADY FLOW MODEL:

Boundary Conditions were defined for the most upstream cross section, using daily flow series data for ELKURU (2009-2016), the time step was varied according to magnitude of flow giving low flow periods computation increment of 12 hours, while high flow periods were calculated using 1-hour computation increment, moderate flow time increments varied between (2-6) hours. For the most downstream cross section, the rating curve for ELHESAI was added as a boundary condition, and for the inline structure, the time series gate opening was defined according to the given rules of operation

Temperature is necessary for calculating falling velocity, Temperature time series were developed for quasi unsteady flow analysis editor. Data for each day was added, ranging from (26° C to 32°C) for the Whole year, regarding its variability corresponding to seasons.

2.3.4 QUASI UNSTEADY FLOW MODEL CONSIDERATIONS:

- 1) Solves the steady flow backwater equations for a series of flows within associated times.
- 2) Gate and reservoir operations are ill-posed so the reservoir elevation will be computed based on the head required to reach steady state equilibrium.
- 3) Quasi-unsteady flow models work in couple with the sediment model giving both invert change and water surface elevation for each cross-section and for a given discharge.

2.3.5 MODEL SENSITIVITY ANALYSIS, CALIBRATION AND VALIDATION:

it worth mentioning that hydrodynamic models are quite sensitive to manning roughness value" n". Initial model runs for manning value (n) of (0.15) s/m^{1/3}, hadn't matched the observed water surface profiles. However, after rigorous trial runs for testing the model with a changing set of governing input parameters, the results had revealed an increase in calculated water level and increase in model accuracy, however, a complicating factor in evaluating channel roughness in sedimentation investigation for alluvial channels is the bed configuration (Simons, 1985), River Nile morphology often exhibits sand dunes formation , therefore, the typical range for manning coefficient is (0.025-0.035) s/m^{1/3} (Simons, 1985), (0.035) s/m^{1/3}

was used e for Merowe reservoir hydrodynamic analysis, and it had produced water surface profiles approximately similar to the recorded water surface profiles for the selected profiles , The model had been run and calibrated after damming for years (2009-2012) then validated for years (2013- 2016).

Nash-Sutcliffe Efficiency coefficient (E), and the root mean square error (RMSE), were used to measure the

accuracy of the calculated water level compared to the observed ones, since the dam started operation at 2009, sufficient data to carry out overall calibration doesn't exist, therefore, calibration was carried out on yearly basis for the period (2009-2012) and validation for the period (2012-2016) , (E) and (RMSE) values for calibration and validation periods illustrated in Table 1.

| CALIBRATION PERIOD | | | VALIDATION PERIOD | | |
|--------------------|------|------|-------------------|------|------|
| Year | E | RMSE | Year | E | RMSE |
| 2009 | 0.83 | 0.53 | 2013 | 0.8 | 0.67 |
| 2010 | 0.85 | 0.72 | 2014 | 0.8 | 0.7 |
| 2011 | 0.82 | 0.62 | 2015 | 0.8 | 0.4 |
| 2012 | 0.85 | 0.72 | 2016 | 0.82 | 0.6 |

Table 1 : Nash-Sutcliffe Efficiency coefficient (E) and Root mean square errors (RMSE) Values for Model

2.4 SEDIMENT MODEL:

To perform sediment transport analysis, the sediment continuity equation had been used to route the sediment from one cross section to the next. Each cross section was made up of a sediment control volume that extends half way upstream and halfway downstream from the cross section.

The transport capacity was calculated for each control volume and compared to the available sediment supply. In general terms, If the supply sediment greater than the transport capacity, deposition occurs. If the supply is smaller than the transport capacity, erosion occurs, both processes occurs as a vertical change, if the supply and transport capacity is equal the channel remains in equilibrium (USACE, February 2016).

Grain size distributions from bed samples had been added to each cross section. Sediment transport was then calculated for each size division separately before added together to a total transported load. The bed gradation was defined as (% finer), using sieve analysis data obtained from laboratory for ELKURU station four classes were obtained at different bed levels (7.28 m, 9 m ,7 m, and 7.65m).

Sediment rating curve was developed for ELKURU station to be set as an upstream boundary condition, the total load was calculated, assuming the bed load represented (20%) of the suspended load.

Equilibrium load was used as an upstream boundary condition, boundary sediment load was computed from bed gradation and sediment transport capacity, these capacities were then introduced as load time series to the next cross section, model changes emphasized on the bed only, isolating bank processes

Sediment routing based on the concept of mass conservation through sediment continuity equation, was computed using Exner Equation (USACE, February 2016):

$$(1 - p)B \frac{\partial n}{\partial t} = - \frac{\partial Q_s}{\partial x}$$

Where:

p = porosity of active layer

B = width of channel (m)

n = Channel Elevation (m)

Q_s = Transported Sediment Load (m³/sec)

x = Distance (m)

t = Time (sec)

from bed processes after a stable hydraulic model was constructed ((USACE, February 2016).

2.4.1 MAXIMUM POTENTIAL SCOURING DEPTH:

From the laboratory analysis, the bed material was defined to be fine sand (0.125 mm-0.25 mm) While the suspended sediment material was defined to be fine silt and the river assumed to be alluvial , it worth mentioning that the most challenging issue in alluvial rivers is to determine the maximum scouring depth , as these rivers have huge potential to scour unlimited depth , some previous studies on Merowe dam had stated a value of maximum scouring depth of 5 meters at the D/S side , however , these studies had been conducted before damming , therefore, the equilibrium of the river had been disturbed , some researchers used an approach to increase the maximum observed scouring depth by a certain

percentage (Jennifer G.Duan, 2008) , For Merowe dam a scouring depth of (7.5 m) had been used for the both boundary conditions , simulating bathymetric changes after 30 years of operation (2009-2038).

2.4.2. SEDIMENT TRANSPORT CALCULATION:

The Following Transport Formulas were checked for their suitability to the defined soil type and classification:

- 1) Laursen Copeland formula (Total Load).
- 2) Yang formula (Total Load).
- 3) Ackre/White Formula (Total Load).

The available data set obtained from Merowe laboratory for bed gradation were added for the cross sections, multi trial and errors were performed and it was found that class 4 represented the soil type better than other classes.Total Sediment was calculated using (Laursen/Copeland, Yang and Ackre/White) formulas. For sorting calculation Thomas Mixing

For inflow input. A time series of 8 years measure at ELKURU (2009-2016) were used to develop 30 years time series by repeating the values, after 30 years of operation (2009-2038) morphological changes had been determined using these repeated values for flow hydrograph.. Recent studies showed that the expected climate changes within the river Nile Basin varying between +-15% for inflow data (Eltahir, 2004) , morphological changes were also addressed considering this changes

2.4.3 SEDIMENT MODEL LIMITATIONS

- 1) Due to the braided features of the Nile River, simulating each flow segment in the model was challenging, because the point Bars and flow paths are in constant change, and the real variability can't be captured by the steady state model.
- 2) The fraction of different grain sizes for the whole reach was not available so the bed gradations from (2008) were generalized for the whole reach.
- 3) The model uses continuity equation and control volume concept to route sediment movement, advection and dispersion concepts aren't introduced in the model.

3. MODEL SENSITIVITY ANALYSIS:

Sensitivity analysis was carried out for the Hydrodynamic Model, The effects of altering different values of manning coefficients for main channel and banks which were studied are (0.03, 0.015 ,0.03) s/m^{1/3} , (0.035, 0.03 , 0.035) s/m^{1/3} and (0.075, 0.06, 0.075) s/m^{1/3} .

Method was used as the most suitable method, the active layer was computed at the beginning of each time step and then adjusted it to the equilibrium depth (maximum potential scouring depth) (USACE, February 2016).

For suspended sediment load calculation , Van Rijn equation was used to calculate fall velocity for the model, Ruby equation was used as an initial guess and then computed a new fall velocity from experimental curves based on Reynolds's number computed from the initial guess (USACE, February 2016).

The model was applied to simulate morphological change after dam has started the operation to obtain changes in river bed profiles before constructing the dam, running the model before damming (2004-2008) showed no significant bed changes. The bed level was determined at each cross section, then the sedimentation process was simulated from (2009 - 2016) using the three equations to compare calculation trends of each equation.

For the sediment model , Laursen and Copeland was used to calculate sediment transport capacity , sensitivity analysis was carried out to study the effect of altering the same manning coefficient values for main channel and banks , also three different sets of time step for high , moderate and low flows were studied Which are: (12,6,2,1) / (6,3,1,0.5) and (3,1.5,0.5,0.25) hours. It should be noted that calibration parameter of critical shields stress had a remarkable effect in the calculated sediment transport capacity, therefore, different values of this parameter Which are (0.039, 0.078 and 0.009) were also used in the sensitivity analysis.

Sensitivity analysis results are represented in figures (4, 5, 6) from the analysis it was found that:

- i. The model was less sensitive to the falling velocity method.
- ii. Using active layer as a sorting method resulted in in-realistic deposition pattern.
- iii. Increasing manning value caused transport capacity to be reduced, and therefore, produced more stable zones of river-bed changes.
- iv. Time step selection was a dominant factor for model stability but it doesn't affect the transport capacity.
- v. The critical shield stress parameter had the most effect on the results, and produced more realistic river bed changes.

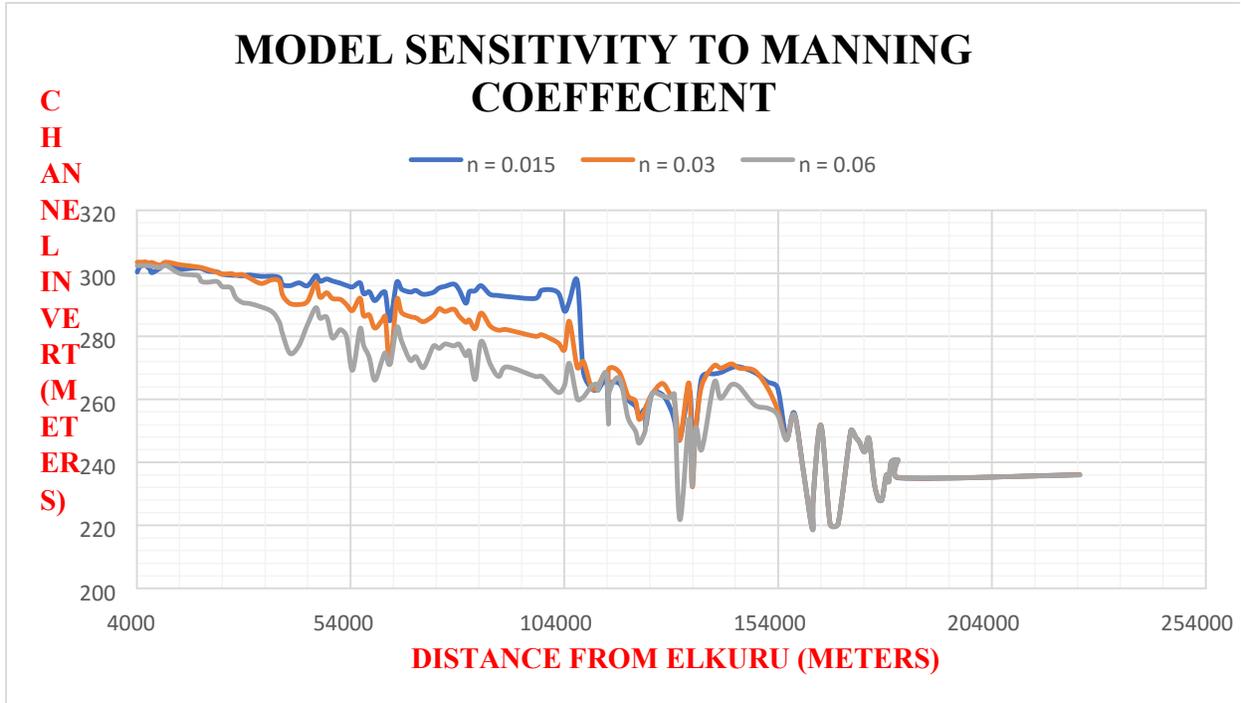


Fig 4: MODEL SENSITIVITY AT DIFFERENT VALUES OF MANNING COEFFICIENT

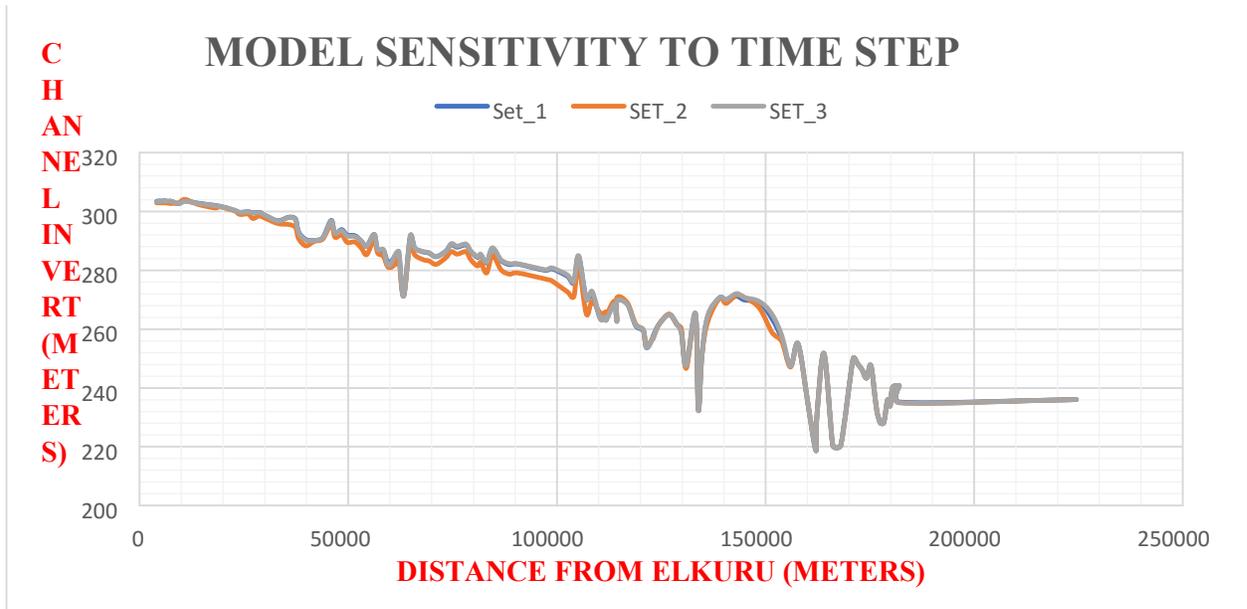


Fig 5: MODEL SENSITIVITY TO TIME STEP

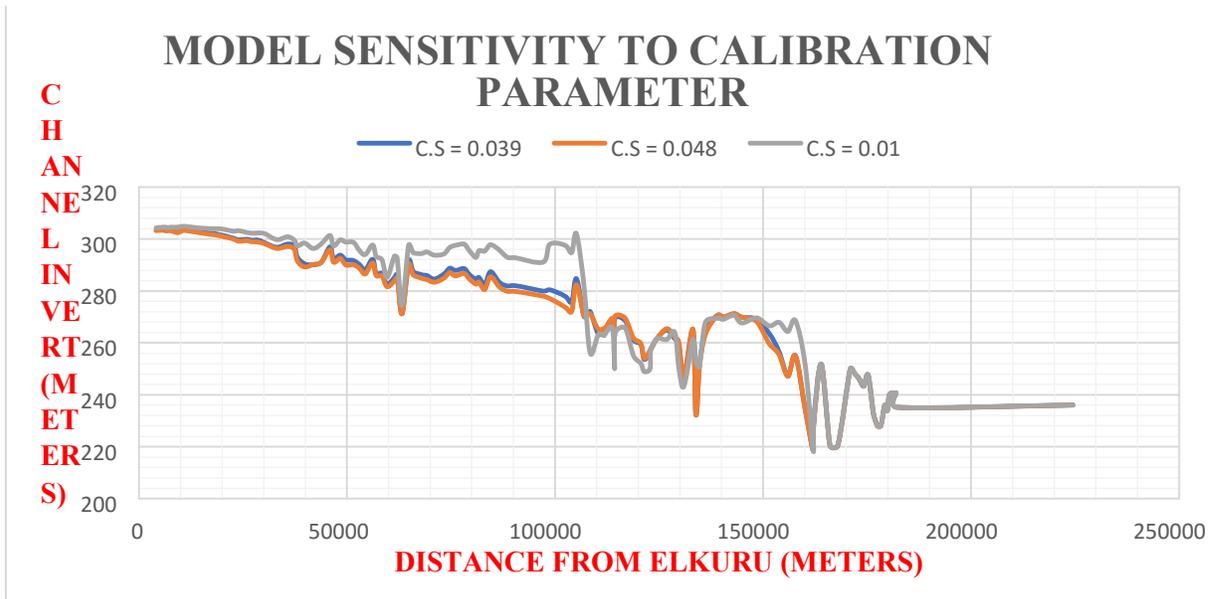


Fig. 6: MODEL SENSITIVITY TO CALIBRATION PARAMETER

**MODEL RESULTS AND ANALYSIS:
4.1 TREND OF SEDIMENT TRANSPORT FORMULAS:**

Using equilibrium load as a boundary condition the trend of sediment transport simulated using the three formulas were compared for 8 years of simulation,

the result of comparison between the three formulas in calculating bed change is presented in fig (7). The three formulas were also compared in terms of volumetric bed changes and the results presented in fig (8).

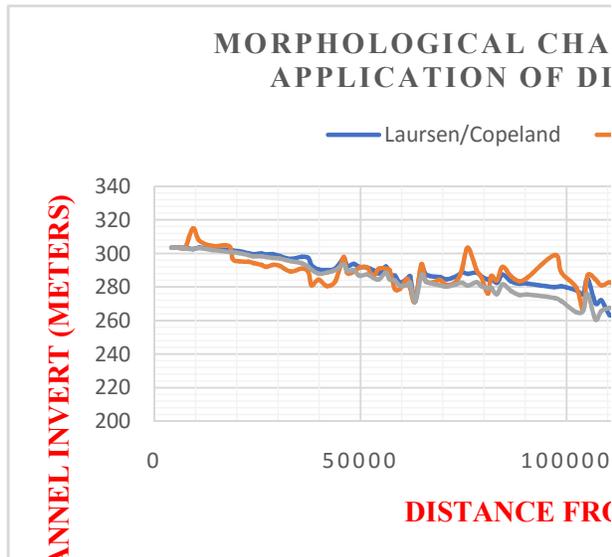
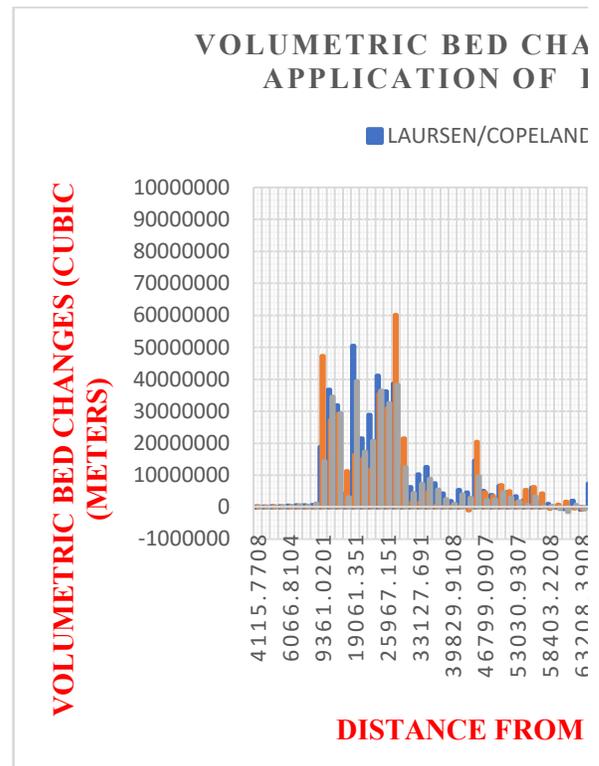


Fig. 7: TREND OF DIFFERENT FORMULAE IN CALCULATING BED CHANGES



NG CURVE AND EQUILIBRIUM LOAD:

It worth mentioning that, applying the numerical model as a predictive one , morphological changes in upstream of Merowe Dam using Laursen/Copeland formula after 8 years of operation (2009-2016) and 30 years of operation (2009-2038) was determined using rating curve and equilibrium load as two different boundary conditions , given two possible

deposition patterns in the reservoir , these changes were obtained for two scenarios , one had assumed

recycling flows for the two types of upstream boundary conditions as shown in fig (9) and fig(10) , the other scenario considered climate changes for drier and wetter climate , drier climate had predicted a decrease by 15% of Nile flows while wetter climate predicted an increase of 15% of Nile flows (Eltahir, 2004) , the morphological bed changes for climate change scenario using equilibrium load as boundary condition are shown in fig (11) and fig (12).

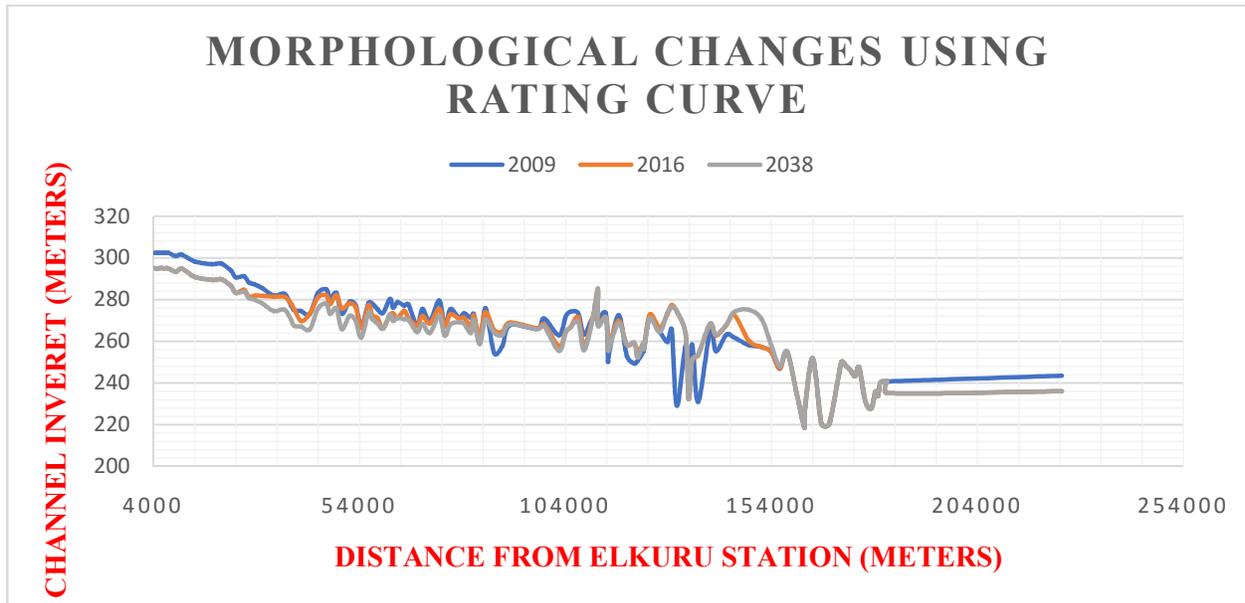


Fig. 9: MORPHOLOGICAL CHANGES USING RATING CUR

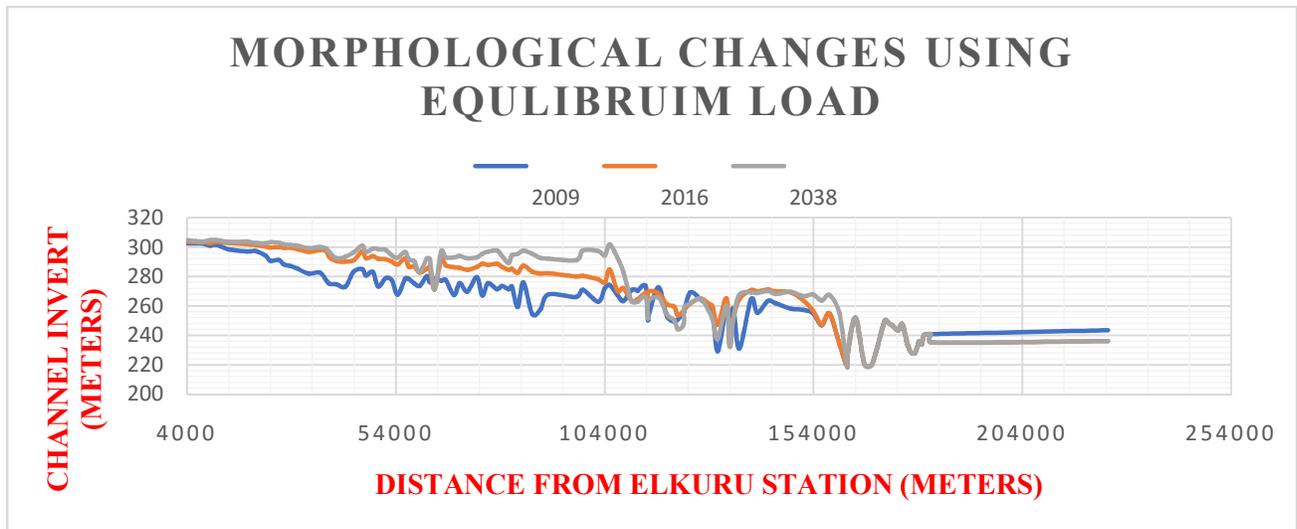


Fig. 10: MORPHOLOGICAL CHANGES USING EQUILIBRIUM LOAD

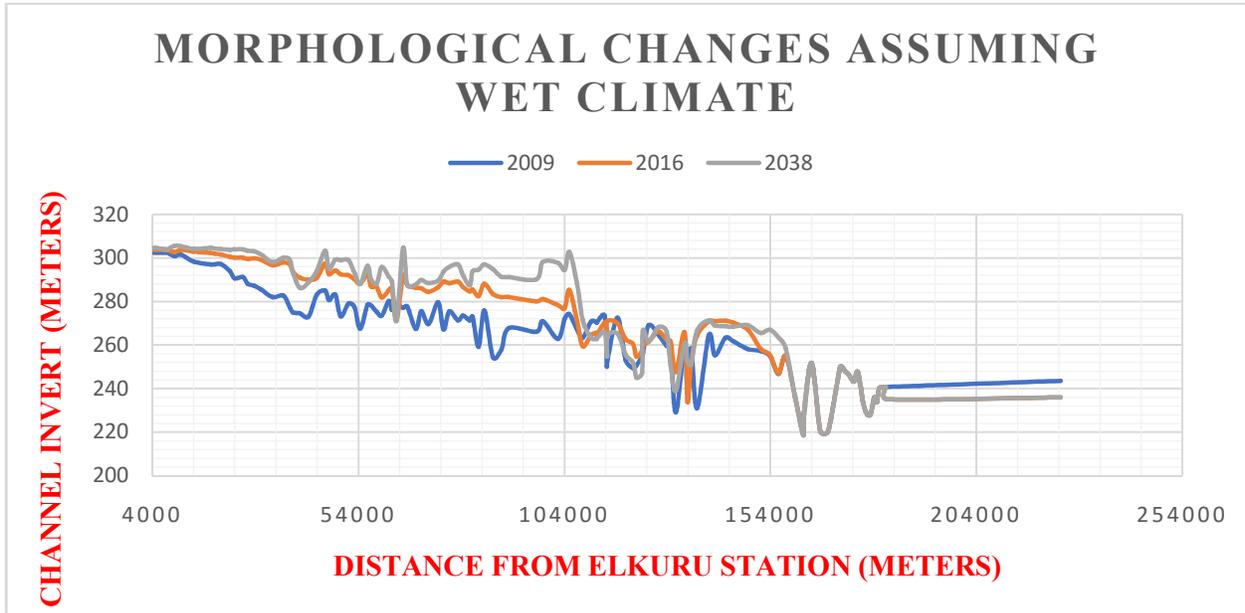


Fig. 11: MORPHOLOGICAL CHANGES ASSUMING WET CLIMATE USING EQUILIBRIUM LOAD

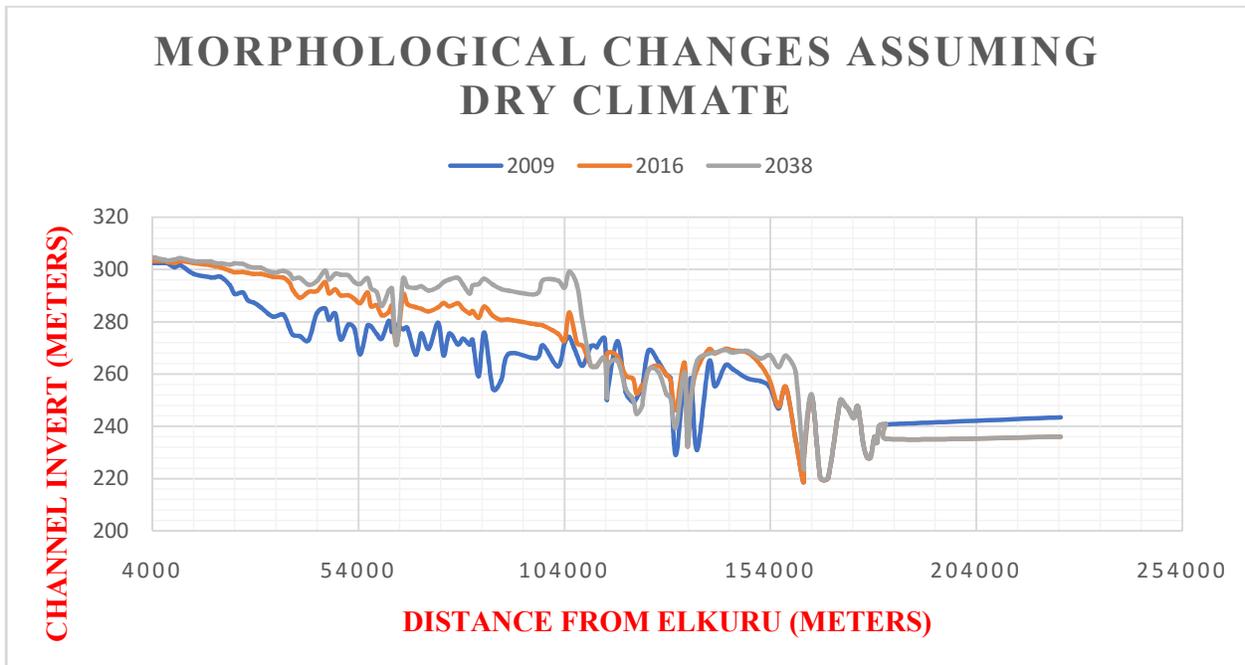


Fig. 12: MORPHOLOGICAL CHANGES ASSUMING DRY CLIMATE USING EQUILIBRIUM LOAD

CONCLUSIONS:

1) For Merowe Dam reservoir sedimentation analysis , river bed changes were calculate for upstream cross-sections after 30 years of operation using Laursen/Copeland to calculate sediment transport capacity , Van Rijin to calculate fall velocity, Thomas mixing method to calculate static armoring and equilibrium load as a boundary condition , and it was found that :

- i) Repeating available inflows at ELKURU station (2009-2016) for 30 years of dam operations resulted in an average of (9.5) m river bed changes and deposition of (5229) Mm^3 , and the average sedimentation rate of (174.3) $Mm^3/year$.
- ii) Increasing available inflows at ELKURU station (2009-2016) by 15% and repeating them for 30 years of dam operation resulted in an average of (9) m river bed changes and deposition of (5602.5) Mm^3 , and the average sedimentation rate of (186.75) $Mm^3/year$.
- iii) Decreasing available inflows at ELKURU station (2009-2016) by 15% and repeating them for 30 years of dam operation resulted in an average of (8) m river bed changes and deposition of (4855.5) Mm^3 , and the average sedimentation rate of (161.85) $Mm^3/year$

2) For Merowe Dam the average change in river bed at delta region after 30 years of operation ranges between (8-10) meters for the three prediction scenarios, which will result in water level by the same amount, therefore, hazard analysis and flood inundation mapping, should be carried out.

3) Using Rating curve as a boundary condition, considered bank stability, modelling both river bed and banks as a movable, the results showed reflected zones of deposition and zones of erosion as well as stable zones along the reservoir length.

4) The expected deposition loss percentage, at the reservoir ranges between (39%-45%) from storage for the three flow scenarios.

5) The three sediment transport capacity formulas were applied to calculate sediment transport capacity and it was found that :

i. Laursen/Copeland and Yang formula produced approximately same results for the river-bed change and cumulative volumetric bed changes.

ii. Ackre/White equation produced larger river bed changes as well as larger cumulative volumetric bed changes, specially at the most upstream cross-sections near ELKURU station.

iii. For the total deposited volume in the reservoir , Ackre/White formula exhibits larger total deposited sediment volumes for the one year of simulation and is expected to be equal to (200) Mm^3 , While both Yang and Laursen/Copelan formulas produced approximately the same volumes for the same simulation period (160 Mm^3) and (165 Mm^3) respectively.

iv. Ackre/White formula exhibits large calculated sediment discharge at ELHESSAI station compared to the other formulas.

5. RECOMMENDATIONS:

1) It worth mentioning that, further studies on the impact of possible new upstream reservoirs, GERD for instance, on sedimentation rate should be judiciously considered.

2) In the future models, to the braided morphology of River Nile can be considered, and it is recommended to apply the law of conservation of mass by modifying cross sections in-order to represent the total sediment load by the sum of the loads of all the braided channels.

3) Methods of Trap efficiency estimation can be introduced to validate modelling of reservoir sedimentation; they can be fitted and modified for Merowe dam Reservoir to represent the nature of reservoir sedimentation for dams in River Nile.

4) It should be noted that the Turbidity and Density Currents may have an effect on reservoir sedimentation, therefore, models to calculate these currents may be coupled with existing models to evaluate the total sediment volumes trapped in the reservoir.

- 5) Adaptation of sediment yield model must be of a great concern for existing and new dams, **GIS** tools such as Soil and Water Assessment Tool (**SWAT**) models can be used to simulate sediment yield and perform different scenarios to assess impacts of changes in land use and historical land use development on sediment yield.
- 6) Climate change as a current global issue must be considered in order to study its effect on sedimentation rate in the reservoir, climate changes models such as Global Circulation Models (GCMs) and National Weather Prediction Models (NWPM) must be coupled with sediment model to initiate a hybrid approach in planning and monitoring reservoir sedimentation.

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